On the Monetization of Mobile Apps

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Abstract

Though the mobile app market is substantial and growing fast, most app providers struggle to monetize apps profitably. Monetizing apps is done in two ways: a) selling advertising space within a free version of the app, and b) selling a paid version, termed *freemium* or *in-app purchase* strategy. In this paper, we present a framework for monetization of mobile apps, using two central empirical regularities concerning the relationship between users and their mobile apps: a) Sampling: While consumers have some prior knowledge of their fit with the app, they remain uncertain regarding their exact utility until they are using it; and b) Satiation: The utility of using the app may decrease with time. While work on the monetization of digital goods has largely overlooked the role of satiation and the consequent retention issues, we show that in combination with uncertainty, it elucidates the role of the segments of consumers that download the free vs. paid version of the app, and how to balance these two segments so as to monetize mobile apps. We encounter two distinct scenarios: In the first, advertising drives most of the revenues; while in the second, revenues are driven by the paid version of the app. We explain how uncertainty and satiation affect the prevalence of the respective scenarios and impact the share of revenues from the paid vs free version of the app. We also demonstrate that an app provider can profit from offering a free version with ads even if advertisers are not paying for these ads. In other words, the app provider benefits from offering a "damaged good" version of the app that includes ads, even if this version is free to consumers, and the advertisers are not paying for the ads.

Keywords: advertising; freemium; mobile apps; online strategy; pricing; retention; satiation

1. Introduction

The number of mobile apps offered to consumers is rising rapidly: By 2018, consumers could choose among nearly six million apps in Google or Apple stores (Statista 2018). Industry reports indicate that over 90% of mobile apps begin as free, and over 90% of the profits from mobile apps come from apps that began as free (AppBrain 2019). Although these apps are "free", market size is notable, at more than $90 billion in 2018 (Freier 2018) and expected to continue to grow rapidly by 2022 (Cheney and Thompson 2018). The largest app category is mobile games, whose revenues are reported to have reached $70 billion in 2018 (Freier 2018), exceeding both video console games and PC games.

Monetizing apps is mainly done in two ways: a) selling advertising space within a free version of the app, and b) selling a paid version, or an in-app purchase strategy that offers additional features for pay (Needleman 2016). As app creators struggle to remain profitable, the question of choosing the correct business model and marketing mix becomes more pressing (Natanson 2016). While advertising and pricing decisions have always been interwoven (Assmus, Farley, and Lehmann 1984), in mobile apps, the interrelationship is magnified significantly due to the specific segments that download each version of the app. Potential users consider adopting the app in its two versions – free, and paid – and make these decisions over time (Gu, Kannan, and Ma 2018). Indeed, many "sample" the free version while considering the paid version (Deng, Lambrecht, and Liu 2018).

To clarify our exposition here, we chose the following example of a popular mobile game of a popular category: Role Playing Games (RPG). RPG is an app category intended for mobile devices, distributed on either Google's Play Store or Apple's iOS App Store as either free, or paid[1]. The game ***Doom and Destiny*** is designed in the 8-bit graphics style of Japanese RPG (JRPG) by Italian developer Heartbit Interactive. The game tells the story of four nerds who get drawn into a fantasy world. Mistaken for heroes, they must battle their way through an adventure in order to defeat a villain. It contains nerdy humor and strong language, so that advocacy group SaferKid recommended 12+ as a safe age to play the game. Launched on Android in 2013, the game currently has over 500k installs and 70k reviews of the free version, and over 50k installs and 18k reviews of the paid version. It is also quite popular, as for example, the paid version has been #9 in the US in the Role Playing category (as of November 2018). In terms of the costs to consumers in either user time or out-of-pocket expenses, in the free version, an ad appears every 3 minutes, covering the entire screen for 5 seconds; while the paid version costs $4.99. The only difference between the two is lack of advertising in the paid version.

In this paper we present a framework for app monetization in this complex environment that involves the following critical factors in new apps' success: ***sampling***, ***advertising***, ***pricing***, and ***satiation.*** We briefly discuss each using the ***Doom and Destiny*** example**.**

Sampling: Although consumers might have some prior knowledge about the app, they remain uncertain about the idiosyncratic utility they can expect from using it (Morvinski, Amir, and Muller 2017). In the above-described example of ***Doom and Destiny***, judging from the reviews, there is uncertainty about a given user's match with the app until s/he begins playing the game. The reason for this might be that the description is rather terse and generic: The user might consider herself a nerd, yet cannot be sure that the humor fits or about the crudity of the language, or some idiosyncrasies of the game's technical aspects. Thus, we find reviews that

[1] The term "free app" is somewhat of a misnomer, as even the "free" version has a cost, i.e., the consumer pays with her time by viewing advertising. However, for convenience' sake, we refer to the ad-based version as "free".

state, "*I didn't expect this to be a gem*…" and "…*love the dialog, cursing included*…", and "…*new way to play with the control setup being something I've never used before*…"

Product fit uncertainty is a major impediment to online markets in general (Hong and Pavlou 2014), even more so regarding consumers' uncertainty about apps, as the majority of apps have been developed by small businesses with unknown reputations (Arora, Hofstede, and Mahajan 2017). Market reports suggest that given the absence of monetary cost to download apps, consumer learning often occurs post-adoption, rather than pre-adoption as in classic consumer markets (Klotzbach 2016). Yet consumers are not entirely in the dark with respect to their match with any given app. For example, some consumers know a-priori that they prefer social games over brain games. However, as each app is unique, even after reading an app's description, consumers are uncertain about the fit between their preferences and the app. As mentioned earlier, over 90% of mobile apps begin as free, and this ubiquity of the free-to-download model of apps is hard to justify without realizing that a considerable number of these downloads are actually sampling done by consumers.

Advertising: The free version of the app is bundled with advertising. Exposure to advertising is irksome to the consumer, arousing annoyance and irritation, and thus the intensity of advertising depreciates the consumer's utility (Wilbur 2008). Advertising creates inconvenience for the consumer in that it effectively reduces the level of content that s/he can enjoy from the app, in addition to inflicting other costs, such as distraction and opportunity cost of time, resulting in a negative effect on utility and retention (Tåg 2009; Goldstein et al. 2014). Indeed, in ***Doom and Destiny***, users have complained about the ads in reviews such as: "…*I'm thinking of buying the full game to get rid of those pesky ads*…" and "*This game is awesome… though there were scientology ads which made confused deeply*. [sic]"

As this scientology advertising demonstrates, app providers tend to have limited control over ad content, which is typically outsourced to ad exchanges that match ads to apps (e.g., Google's Marketing Platform; Yahoo's Flurry; OpenX). However, the app provider can select the level of advertising intensity, and thus Heartbit Interactive, for example, can decide to display an ad every 2 minutes rather than every 3 in ***Doom and Destiny***. These additional ads come with a cost, though: Ad exchanges pay developers on a cost-per-click basis. Posting more ads leads to lower click-through rates, leading to lower ad conversion rate, and thus to lower revenues from advertising (Babu 2018). Moreover, given the strong resentment of many individuals toward ads in games (Lagace 2018), heavy advertising can harm the brand name such that it interferes with firm's ability to acquire customers and retain them. So there is a cost to excessive advertising.

Price: Consumers may choose to eliminate the annoyance of ads, for a price: The app provider offers a paid version of the app. If consumers choose to pay this price, the ads are removed and the consumers enjoy the app's full utility. Hence Heartbit Interactive can decide to price ***Doom and Destiny*** at $3.99 rather than the current $4.99. As we show later, choice of price and advertising levels has a profound effect on the size and revenues of the consumer segments that download the paid vs the free version of the app.

Satiation: Mobile app retention rates are typically much lower than observed retention rates for classic products: Industry reports suggest that across all categories, over 70% of app users churn within 90 days (Perro 2018). This introduces additional complexity into the decision making, as the user who is targeted by advertising might have churned already in early periods of the app life cycle. At the individual level, marketers realize that churn is closely related to declining engagement over time, which lowers users' utility from apps (Adjust 2018). From a behavioral standpoint, such decline is consistent with abundant evidence of satiation's role in products' consumption patterns, hedonic products in particular (Galak, Kruger, and Loewenstein 2013; Sevilla, Zhang, and Kahn 2016; Galak and Redden 2018). ***Persistence*** is the opposite of satiation, as churn is to retention: the higher the persistence, the lower the satiation and the more utility remains by the second period. Note that satiation reflects changes in utility over time.

It is thus not surprising that satiation has been found to affect consumption behavior for mobile apps such as games (Han, Park, and Oh 2016; Hui 2017). However, the extent of the phenomenon may differ considerably between app categories, which likewise vary considerably in their retention rates (Adjust 2018). The app research firm Flurry, for example, found that the high-retention categories are health and fitness, weather, magazines, and business and finance; while action games, sports, family, entertainment, and puzzles have much lower retention probabilities (Klotzbach 2016). This difference may be thus related to the extent to which apps are perceived more as hedonic vs utilitarian (Schulze, Schöler, and Skiera 2014).

In our example, while we do not have access to ***Doom and Destiny*** usage data, we can find out the mobile games category's average usage. In order to assess the level of persistence, we make two assumptions[2]: First, a "period" is a week; and second, usage is a reasonable proxy for utility. Under these assumptions, we utilize an Adjust Mobile Benchmark Q3 2016 study (the latest available) to compute average usage, in minutes per day, of the first and second week for Android users. This if we use the average usage time as a proxy for utility, then the average time a user spends on a game is 15.3 minutes per day in the first week, and 8 minutes per day in the second, leading to a persistence parameter $\frac{8}{15.3} = 0.53$.

[2] As this is an example for computation of satiation only, the assumptions are not inherent to our analysis, and are used solely for this example. Nonetheless, both assumptions are reasonable: As industry reports indicate that the time frame used for these fast-moving games is either daily or weekly, with 90% churn after 14 days; and usage, while not perfect, might be a good proxy for the engagement and therefore utility enjoyed by the player.

Overall, we show how monetization policy creates the market reality of paid and free apps, taking into account the joint effect of customer uncertainty and satiation. The equilibria in our setting result from consumers maximizing utility by choosing free or paid versions, and the app producer maximizing profits by optimally choosing price and advertising. We see that this setting in fact leads to two distinct scenarios: In the first, advertising drives most of the revenues; while in the second, revenues are driven by the paid version of the app. We will demonstrate how uncertainty and satiation affect the prevalence of the respective scenarios.

In particular, our analysis elucidates the importance of satiation in the market. Satiation's role is notable given that academic discussion of app monetization to date has largely overlooked the relevance of user duration with the app. We show that the optimal prevalence of free versus paid apps is related to level of satiation: When satiation increases, initially, the paid version's share of the app decreases; then, when satiation is high enough, the app provider offers only the ad-based version of the app. The following analysis helps us to understand how and why this happens, and how persistence also creates value through its effect on advertising and price.

We also show that the app provider benefits from offering a free version of the app that contains ads, even if advertisers are not paying for these ads. The logic behind this "damaged good" is that the free version acts as a sample that enables consumers to learn their fit with the app. The app provider would rather add advertising to this version in order to keep the consumers who derive high value from the app away from the sample, and push them toward the paid version. Overall, our analysis demonstrates how the respective shares of paid and free apps are a function of a complex process that combines app features and the marketing mix to yield user utility. Managers need to understand this process in order to set realistic expectations, understand the impact of various parameters, and design strategies to increase profitability.

2. Related literature

Our work is relevant to research efforts aimed at understanding firm behavior in free digital markets. Research in this area has largely focused on the choice between content and advertising in the context of media markets, where the basic tradeoff is that moving from an advertising-only revenue model to charging for content, may reduce usage and thus hurt advertising revenues (Lambrecht et al. 2014). Specifically, our work is related to recent calls for better understanding of freemium product design toward maximizing customer lifetime value (Kannan and Li 2017).

Empirical results regarding the consequences of such a choice vary. Profitability in such contexts may depend on factors such as the type of promotions used (Pauwels and Weiss 2008), temporal changes in demand (Lambrecht and Misra 2016), factors that affect user engagement (Rutz, Aravindakshan and Rubel 2019) and users' ability to bring in new business through referrals (Lee, Kumar, and Gupta 2017). It has also been suggested that the existence of free apps may affect the speed of paid alternatives' growth (Arora, Hofstede, and Mahajan 2017; Deng, Lambrecht, and Liu 2018). Analytically, noticeable effort has centered on the questions of profit from content vs. advertising in two-sided media markets such as newspapers (Halbheer et al. 2014). The tradeoff between paid content and advertising may depend on competitive intensity (Godes, Ofek, and Sarvary 2009), consumer heterogeneity in willingness to pay (Prasad, Mahajan, and Bronnenberg 2003), or the extent to which consumers dislike advertising (Tåg 2009).

Viewing the use of a free product as a sampling mechanism, recent work has suggested that advertising's effectiveness, coupled with consumers' expectations regarding quality, can determine which revenue source firms should focus on in attempting to enhance profitability (Halbheer et al. 2014). Moreover, it has been shown that the appeal of the free option is influenced by the effectiveness of word of mouth (Niculescu and Wu 2014), and by quality and other design parameters (Li, Jain, and Kannan 2019). These research efforts, however, do not consider customer satiation's role and its effect on retention in the subsequent market state.

Our work is also relevant to the literature on product line design (Moorthy 1984; Moorthy and Png 1992; Kannan 2013; Biyalogorsky and Koenigsberg 2014; Gu, Kannan, and Ma 2018). The main driving force behind product line design and pricing is the fact that consumers derive differing valuations from the product's attributes. The firm knows the distribution of preferences, yet not individual preferences, and thus cannot engage in standard monopoly (first-degree) price discrimination. Instead, it offers a line of products and relies on self-selection of consumer segments thereto, thereby permitting partial discrimination among consumers. This self-selection process relies on incentive-compatibility constraints that deter one segment from purchasing the "wrong" product that was designed for another segment.

As Gu, Kannan, and Ma (2018) demonstrated, questions of app market planning, for example extending the premium product line, can be analyzed from a product line standpoint. Indeed, our model features this type of price discrimination, where the firm offers two product versions (paid, and free), and consumers self-select, where the low-valuation consumers download the free version (with the nuisance of ads), and the high-valuation consumers choose the paid version. What complicates the analysis considerably is the introduction of uncertainty: As Biyalogorsky and Koenigsberg (2014) indicated, uncertainty complicates the design problem of developing products that satisfy the incentive-compatibility constraints. In our case, the firm has to design both versions, knowing full well that some consumers will sample the product and decide about purchasing either version in the subsequent period, once uncertainty is resolved.

Given our emphasis, our work is relevant to the avenue of research that addresses customer retention's consequences (Libai, Muller, and Peres 2009; Ascarza et al. 2018), and how optimal retention efforts may differ based on market characteristics (Musalem and Joshi 2009; Shin and Sudhir 2012; Subramanian, Raju, and Zhang 2013). Regarding online behavior, researchers have considered the antecedents of users' tendencies to stick with an app (Hsu and Lin 2016). Yet the dynamics of how satiation, and consequently retention, affect monetization in free digital markets, have not been examined.

Finally, as the consumers in our setting learn about the app, our work addresses the demand-side literature on consumers' learning (Erdem et al. 1999; Iyengar, Ansari, and Gupta 2007). Our learning structure is not on specific attributes of the app, but rather on the idiosyncratic utility that the consumer can expect from experiencing the app, similar in spirit to Ackerberg (2003). Free samples in technological products have been shown to be an effective way to convey information, lowering consumers' risk associated with adoption, and consequently accelerating adoption (Foubert and Gijsbrechts 2016; Li, Jain, and Kannan 2019). Here we show how such learning combines with post-adoption satiation to affect market outcomes.

3. The model

We begin by describing the factors affecting consumers' utility, then proceed with the app provider's choice of advertising and pricing, and conclude with the formal utility maximization problem of the user.

As some consumers download the free version and some the paid version, we need to capture this heterogeneity. Thus the base utility of a consumer, denoted by α, is distributed uniformly in the interval (0,1). To capture sampling, we need to introduce uncertainty into the model. We let uncertainty be additive, in that each consumer is assigned a base utility of α, and then nature either adds or subtracts ε to or from α (with probability of 1/2 to each option). This process is common knowledge, and each consumer knows her own α, yet does not know whether nature added or subtracted ε in her case. Thus the utility is $\alpha - \varepsilon$, or $\alpha + \varepsilon$, where $\varepsilon \geq 0$, and is distributed uniformly in the interval (0,1). Thus, the parameter ε represents the degree of uncertainty that the consumer faces with respect to her true base utility. This uncertainty is resolved when using the app, irrespective of whether she uses its free or paid version. Note that though both ε and α are distributed on (0,1), these distributions represent two distinct phenomena: The former is the source of uncertainty that is needed to model sampling, i.e., without uncertainty, there is no need for sampling. The latter is needed to model heterogeneity in the user population that we observe in reality: Some users download the free version and some the paid version, and they adopt the app at different times.

To convert advertising levels (denoted by γ) to profits, we define a conversion parameter as follows: As the price is expressed in monetary terms such as dollars or renminbis, we need a parameter, denoted by k, to convert advertising intensity γ to the same monetary term. The advertising parameter k thus represents the monetary payment received by the app provider from advertisers per unit of ad intensity. It thus measures the advertising's effectiveness, which might be a function of the nature of the app developer's audience.

Consistent with the customer duration literature that highlights retention rather than churn, we define the fraction of the utility that is still experienced in the second period, and label this parameter (denoted by δ) the *persistence* parameter. Persistence is the opposite of satiation, as churn is to retention: the higher the persistence, the lower the satiation and the more utility remains by the second period. Table 1 summarizes the models parameters:

Table 1: App monetization model's parameters

Parameter	Description	Comments
α	Heterogonous base-utility level	α is distributed uniformly, $0 \leq \alpha \leq 1$
δ	Persistence parameters	$\delta \leq 1$ high δ implies more utility retained in second period
p	Price of the app	Price is paid once in first period, even if app is used in second period
γ	Advertising intensity	$\gamma \leq 1$ Utility is decreased by γ when using free app
ε	Uncertainty parameter	ε is distributed uniformly, $0 \leq \varepsilon \leq 1$ utility is $\alpha - \varepsilon$, or $\alpha + \varepsilon$ (with probality ½)
k	Advertising parameter that converts ad intensity to monetary terms (e.g., € or ¥)	monetary payment received by the app provider from advertisers, per unit of ad intensity

The app provider chooses the price and level of advertising intensity so as to maximize its profits from the two periods. We normalize the overall market potential to 1. Let P_1 and P_2 be the share of consumers who use the ***paid*** version of the app, and A_1 and A_2 the share of consumers who use the ***free*** version of the app, in Periods 1 and 2 respectively.

As we discussed earlier, there are costs associated with advertising, which we assume to be convex. These convex costs are equivalent to assuming diminishing marginal returns to investment in advertising (Pindyck 1982; Vakratsas and Ambler 1999)[3]. The objective function of the app provider is thus:[4]

[3] While we could have added a parameter ω to convert advertising costs into monetary terms, it is straightforward to show that ω is just a scale parameter that changes both decision variables at the same proportions, and does not have any effect on the scenarios presented next. We thus, without loss of generality, assumed that $\omega = 1$.

[4] While consumers who buy the paid version in the first period may or may not stay for the second period, they do not affect the profit function, as these consumers do not generate any revenues in the second period. Thus, consumers pay for the app in the second period only if they switched from the free version to the paid one in that period.

(1) $\Pi(p,\gamma) = p(P_1 + P_2) + k\gamma(A_1 + A_2) - \gamma^2$

The profit is a function of the two decision variables of the app provider – price, and advertising intensity – which of course depend on the model's parameters: k, ε, and δ. Following common practices and contractual settings in actual app markets, we assume that price and advertising intensity do not change in the second period. Specifically, in the real app market, the price and ad intensity are embedded in the software and are rarely changed. For example, the business press reports that changing the in-app purchase price of an app is a complicated, resource-intensive process that can intimidate developers (Ogg 2013). The complexity of price and advertising changes is also driven by apps' relatively short life cycles, and by the fact that advertising contracts are entered into in advance with third parties.

4. Consumer's expected utility

Table 2 depicts the utility levels of the individual, depending on her choice and time period:

Table 2: Utility levels of the consumer[5]

	Time Period 1	Time Period 2	Comments
Consumer does not use the app	0	0	
Consumer uses the free version	$\alpha(1-\gamma)$	$\alpha(1 \pm \varepsilon)(1-\gamma)\delta$	Utility in the second period is lower ($\delta \leq 1$)
Consumer uses the paid version	$\alpha - p$	$\alpha(1 \pm \varepsilon)\delta$	Price is paid only once

Thus utility is multiplied by $1-\gamma$ if the free version is used, and by δ in the second period. In addition, because of usage in the first period, the consumer realizes her fit to the app, and her

[5] The utility from adopting only in Time Period 2 is lower than that of adopting in the first period, thus if it was worthwhile adopting in Time Period 2, it is also worthwhile adopting in Time Period 1.

utility is increased or decreased by ε. Note that this table summarizes the utilities that the consumer takes into account while choosing between the alternative, which we formalize next.

Consumer chooses paid version. We begin by considering the case in which the user chooses the paid version in the first period. In such a case, she has two options in the second period: (a) continue to use the paid version or (b) discontinue using the app. The option of using the free version in the second period is not appealing to her, as it contains ads, which decrease her utility, and she has already paid for the ad-free version. Therefore, the choice between using the app or not, which maximizes her utility, can be expressed as follows:

1. The consumer, regardless of her intrinsic valuation, uses the paid version of the app in the first period (first term of Equation 2).
2. If $\alpha < \varepsilon$, she will use the paid version in the second period only when her utility realization is positive, with probability ½ (second term of Equation 2).
3. If $\alpha > \varepsilon$, the user will use the app in the second period irrespective of her utility realization (third term of Equation 2).

Accordingly, her expected utility is given by:

(2) $$U_p(\alpha) = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon)I\{\alpha < \varepsilon\} + \alpha\delta I\{\alpha > \varepsilon\}$$

Consumer chooses free version. Next, we consider the case in which the user chooses the free version in the first period. In such a case, he has three options in the second period: a) discontinue use; b) keep on using the free version; or c) pay for the premium version. Therefore, the choice between using the app or not can be expressed as follows:

1. the consumer, regardless of his intrinsic valuation (α), uses the free version in the first period (first term of Equation 3).
2. If $\alpha < \varepsilon$, he will use the app in the second period only when his utility realization is positive, with probability ½ (second term of Equation 3).
3. If $\varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon$, he will use the free version in the second period irrespective of the utility realization (third term of Equation 3).

4. If $\alpha > \frac{p}{\gamma\delta} - \varepsilon$, he will buy the app in the second period in case his utility realization is positive, and use the free version if the realization is negative (last term of Equation 3).

Accordingly, his expected utility is given by[6]:

$$(3)\ U_a(\alpha) = \alpha(1-\gamma) + \frac{\delta}{2}(\alpha+\varepsilon)(1-\gamma)I\{\alpha < \varepsilon\} + \alpha\delta(1-\gamma)I\left\{\varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon\right\}$$

$$+\frac{1}{2}\left[\left((\alpha+\varepsilon)\delta - p\right) + \delta(\alpha-\varepsilon)(1-\gamma)\right]I\left\{\alpha > \frac{p}{\gamma\delta} - \varepsilon\right\}$$

5. Equilibrium scenarios

When considering the equilibria that result from consumers maximizing their utility by choosing the free vs paid version in the two periods, and the producer maximizing profits by optimally choosing price and advertising intensity, we encounter two distinct scenarios that are summarized in Table 3.

Table 3: Ad-based vs. paid-based scenarios[7]

scenario	**% revenues from paid users**	**% paid users**	**Prevalence** (% of all cases)	**average *k***	**average ε**	**average δ**
Ad-based: High share of ad revenues & free users	24%	17%	77%	0.56	0.40	0.48
Pay-based: High share of paid revenues & paid users	64%	45%	23%	0.30	0.83	0.56

[6] For this equation and the rest of the analysis, we need both γ and δ to be strictly positive (positive δ for some of the cases).

[7] Note that a) the first three columns contain figures that are outputs of the equilibrium, while the last three contain the inputs that lead to the equilibrium, and b) the partition into pay vs. ad-based scenarios is based on the results, i.e., after calculating the profit, we grouped together the cases that lead to the two rows of the table; and c) the partitions into cases were continuous, in that they gradually switched from one case to the next. Thus for example, all cases in which $\varepsilon \leq 0.44$ resulted in the ad-based scenario. Cases in which $\varepsilon \geq 0.45$ resulted in gradual increase in the number of pay-based cases as the parameter *k* decreased from one to zero.

The scenarios in Table 3 are divided into two distinct groups: In the first, advertising drives most of the revenues, while in the second, revenues are driven by the paid version of the app (first column of Table 3). Note that regardless of whether revenues are driven by advertising or the paid version, the majority of users of the app download the free version (second column of Table 3). The two scenarios also differ substantially in their prevalence, as depicted in the third column of the table, in that the ad-based scenario constitutes more than three quarters of all cases. In addition, the scenarios include one additional special case each, to be discussed next.

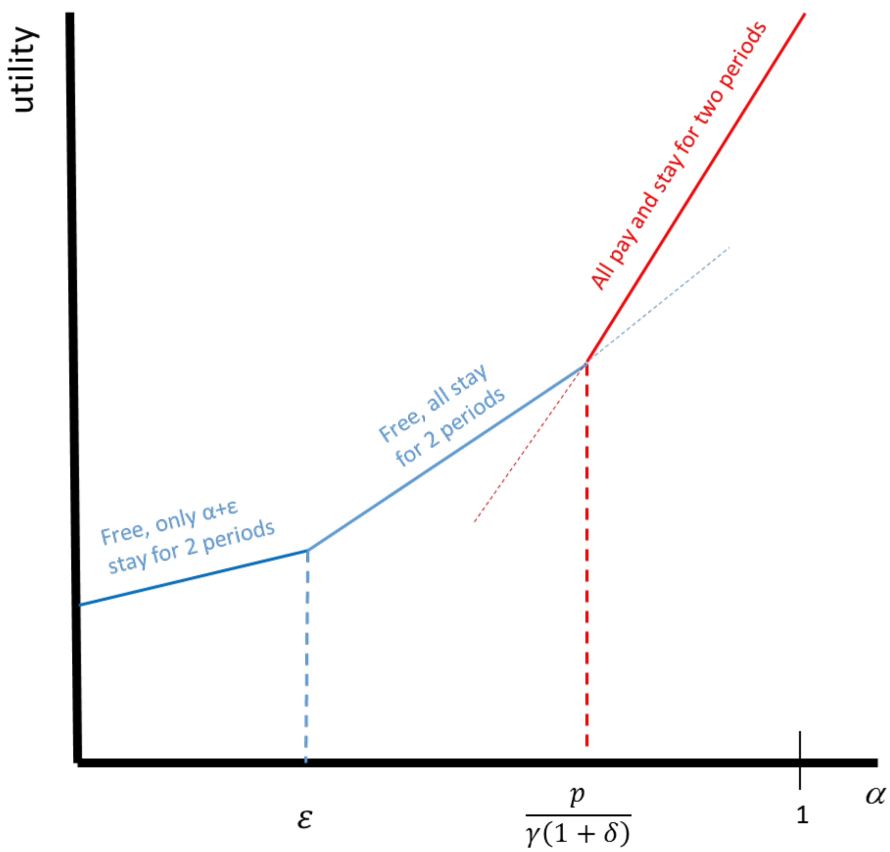
Observing the average parameter values for each scenario, the ad-based scenario is characterized by high advertising effectiveness (k), and thus high levels of advertising, and relatively low uncertainty (ε). The reason for high k is obvious: higher advertising effectiveness implies better and more efficient use of resources devoted to advertising, and, thus ceteris paribus, advertising will be higher, and more revenues will be yielded by this more efficient tool. With respect to uncertainty, consumers who use the ***paid*** version are betting on a positive outcome of the utility realization. If this happens, they will enjoy the full benefit of this occurrence, as they do not pay for the app in the second period. Thus under higher uncertainty, consumers will tend to use the paid version of the app. Lastly, though the difference is not large, the same applies to satiation, which implies higher second-period utility, ergo higher likelihood of choosing the paid version.

The analysis is based on analytical solutions of optimal price, advertising, and profits, in addition to supplementary numerical simulation as follows: The resultant profit function, when plugging in the optimal levels of price and advertising, is too complex for the analysis that compares profits from various scenarios. Thus we calculated the value of the profit function, plugging in a different set of parameters and conducting the analysis numerically based on these calculations. Specifically, we used a full factorial design to generate sets of parameters for 101 levels (from zero to one) for each parameter ($\boldsymbol{k}$, $\boldsymbol{\varepsilon}$, and $\boldsymbol{\delta}$), resulting in $101^3 = 1{,}030{,}301$ cases. These calculations enabled us to determine which parameters lead to a scenario in which most of the profit comes from ads vs pay, and then characterize these differing scenarios.

5.1 Ad-based scenario

This scenario, as well as the pay-based scenario, is best explained via figures that enable us to derive the demand for the app from its two segments: paid, and free. Figure 1 presents the relationship between the intrinsic utility α and the resulting utility.

Figure 1: Demand derivation, ad-based scenario



The "high" segment, denoted by "all pay and stay for two periods" in Figure 1, represents the users whose intrinsic utility α is high enough that it is optimal for them to download the paid version and stay for two periods. From Equation 2 it is clear that the user's utility is $\alpha - p + \alpha\delta$ where the first term $(\alpha - p)$ is her utility in the first period, while the second term $(\alpha\delta)$ summarizes her utility in the second period. If her intrinsic utility α is slightly lower, then s/he will download the free version and stay for two periods with resultant utility of $\alpha(1-\gamma) + \alpha\delta(1-\gamma)$, as per Equation 3. It is straightforward to check that these two lines intersect when $\alpha = p/\gamma(1+\delta)$. Accordingly, it is easy to check that the two blue lines intersect when $\alpha = \varepsilon$.

Thus, we have three segments of consumers of differing sizes:

1. For high α, users download the paid version and stay for two periods. The size of this segment is $1 - p/\gamma(1+\delta)$.
2. For intermediate α, users download the free version and stay for two periods. The size of this segment is $p/\gamma(1+\delta) - \varepsilon$.
3. For low α, users download the free version and only those whose utility realization is positive (+ε with probability ½) will stay for two periods. Their size is thus $\varepsilon/2$.

When we add these segments, each multiplied by price or advertising as per Equation 1, we obtain the profits for the app in the two periods as follows:

$$(4)\ \Pi(\text{ad}-\text{based scenario}) = p\left(1 - \frac{p}{\gamma(1+\delta)}\right) + k\gamma\left(\frac{p}{\gamma(1+\delta)} - \frac{\varepsilon}{2}\right) - \gamma^2$$

To obtain optimal price and advertising, we differentiate Equation 4 with respect to p and γ and equate to zero. See Appendices A and B for details[8].

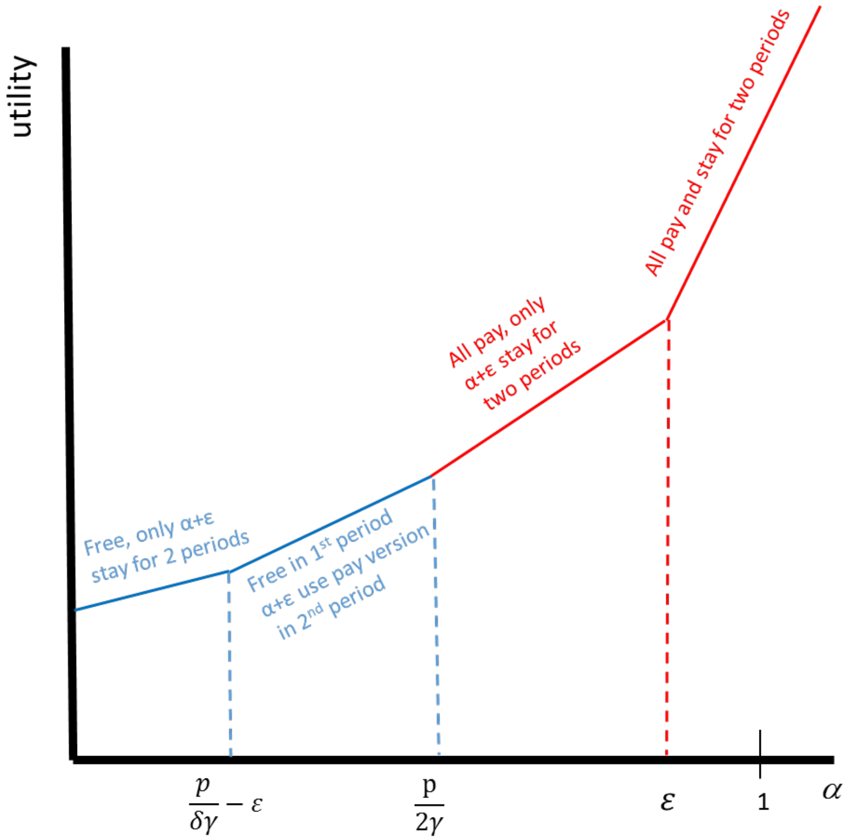
There is a special case of this scenario in which the intersection point of the paid and free versions (at $p/\gamma(1+\delta)$) is greater than or equal to 1 ($p/\gamma(1+\delta) \geq 1$). Thus in this case we're left with only two segments (2 and 3 of the list above). The rest of the analysis follows through in the same manner, *mutatis mutandis*. See Appendices A and B.

[8] It is straightforward to check that for both Equations 4 and 5, the Hessian matrices are negative semidefinite, and thus second-order conditions for maximization of the profit functions are indeed satisfied.

5.2 Pay-based scenario

In the same manner as in the ad-based scenario above, we can construct Figure 2 with its four segments:

Figure 2: Demand derivation, pay-based scenario



First note that as ε is high, all users whose intrinsic utility (α) is lower than ε and whose utility realization is negative (-ε with probability ½) will not use the app in the second period. This includes all segments except for the first.

1. For high α, users download the paid version and stay for two periods. The size of this segment is $1 - \varepsilon$.
2. For intermediate-high α, users download the paid version and only those whose utility realization is positive (+ε with probability ½) will stay for two periods. The size of this segment is thus $(\varepsilon - \frac{p}{2\gamma})$.

3. For intermediate α, users download the free version and those whose utility realization is positive switch to the paid version in the second period. The size of this segment is $\frac{p}{2\gamma} - \frac{p}{\delta\gamma} + \varepsilon$ for the free version, and half this number for the paid version.
4. For low α, users download the free version and only those whose realization is positive will stay for two periods. Their size is thus: $(\frac{p}{\delta\gamma} - \varepsilon)/2$.

When we add these segments, each multiplied by price or advertising as per Equation 1, we obtain the profits for the app in the two periods as follows:

$$(5)\ \Pi(\text{pay} - \text{based scenario}) = p\left(1 - \frac{p}{2\gamma}\right) + \frac{p}{2}\left(\frac{p}{2\gamma} - \frac{p}{\delta\gamma} + \varepsilon\right) + k\gamma\frac{p}{2\gamma} + \frac{k\gamma}{2}\left(\frac{p}{\delta\gamma} - \varepsilon\right) - \gamma^2$$

In order to obtain optimal price and advertising, we differentiate Equation 5 with respect to p and γ, and equate to zero. See Appendices A and B for details.

From Figure 2 it is clear that for Segment 3 to exist (users who download the free version and whose utility realization is positive, and switch to the paid version in the second period) requires that $\frac{p}{\delta\gamma} - \varepsilon \leq \frac{p}{2\gamma}$. There is a special case of this scenario in which the reverse inequality holds, and thus we're left with only three segments (1, 2, and 4 from the list above). The rest of the analysis follows through in the same manner, *mutatis mutandis*. See Appendices A and B.

6. Main results

In this section, we present the main results of our analysis beginning with the effects of the model's parameters on the app provider's optimal choices in terms of price and advertising[9].

[9] Some of the results were obtained analytically and some numerically, as follows: Result 1 and Table 4: Analytical for the ad-based scenario and numerically for the pay-based. Results 2 and 3 are all numerical, and Result 4 analytical with numerical verification for the conditions for pay- vs ad-based scenarios (see Appendices B and C).

6.1 Price and advertising sensitivities

With our analytical solutions, we can derive optimal decision variables' sensitivity as a function of the model's parameters in each of the cases: ad-based, and pay-based. The results are given in Appendices A and B and presented in Table 4 and Result 1.

Table 4a: Ad-based scenario: Effects of advertising effectiveness, uncertainty, and persistence*

	Sensitivity to ad effectiveness (k)	**Sensitivity to uncertainty (ε)**	**Sensitivity to persistence (δ)**
Advertising (γ)	Strictly Positive	Negative	Positive
Price (p)	Positive	Negative	Positive

Table 4b: Pay-based scenario: Effects of advertising effectiveness, uncertainty, and persistence

	Sensitivity to ad effectiveness (k)	**Sensitivity to uncertainty (ε)**	**Sensitivity to persistence (δ)**
Advertising (γ)	Strictly Positive	Strictly Positive	Positive
Price (p)	Strictly Positive	Strictly Positive	Strictly Positive

* The sensitivity is strictly positive if for all values of the parameters, the derivative is strictly positive. It is positive if the derivative is positive, yet a combination of the parameters might make it zero, or if in one of the special cases it is zero; similarly for negative.

Result 1: *Optimal price and advertising increase in ad effectiveness and persistence in both scenarios. They increase in uncertainty in the pay-based scenario, and decrease in uncertainty in the ad-based scenario.*

The reason that advertising and price increase with an increase in persistence (δ) is because such increase makes the user's utility in the second period higher, ergo her willingness to pay increases. This willingness to pay can materialize in one of two ways: She is willing to pay more for the app, or she is willing to be exposed to more advertising while using the app. The reason that advertising and price increase with an increase in advertising effectiveness is that such increase induces the app provider to increase the level of advertising. Given the increase in ad intensity, the paid version becomes more appealing, thus enabling the app provider to charge a higher price.

The rationale behind uncertainty's increasing effect on price and advertising is a bit different: At first glance, it appears as if ε's role is symmetric, as it is represented by $\alpha - \varepsilon$ or $\alpha + \varepsilon$ in the consumer's utility. Yet this is misleading as, if the utility realization is $\alpha - \varepsilon$, then the consumer might not use the app at all in the second period; while if the utility realization is $\alpha + \varepsilon$, the consumer might either continue to use the free app, or switch to the paid version.

As explained in Figure 1, in the ad-based scenario, an increase in uncertainty causes Segment 3 to increase at the expense of Segment 2, that is, the number of those who stay increases only if their utility realization is positive at the expense of those who stay regardless of their utility realization. The net effect is thus negative, causing the app provider to lower the costs to consumers in terms of price and advertising.

As shown in Figure 2, in the pay-based scenario, uncertainty is high, and all users whose utility is lower than ε and whose realization is negative (-ε with probability ½) will not use the app in the second period. This includes three of the four segments. Thus when uncertainty increases, there is positive asymmetry, as most users would not have used the app in the second period already. Now that uncertainty has increased, these users' utility increases in the second period, causing the app provider to increase costs to consumers in terms of advertising and price.

6.2 Share of revenue shares from free vs paid versions of the app

As we discussed in Section 4.1, there is a special case of an ad-based scenario in which there are only two segments, both downloading the free version. Thus, the paid version of the app is not made available to the users. This is a relatively large case, or about 25% of all cases. To better understand the reasons behind the app provider's (optimal) decision to offer a free version only, we note that this sub-scenario is characterized by a very low persistence parameter level (average δ = 0.33) implying high customer satiation, and relatively high advertising effectiveness (average k = 0.84). When persistence is low, second-period utility is low, and many consumers cannot see a justification in paying for the app when it is highly unlikely that they will stay for a second period. Thus, the company focuses on the free version, wherein more consumers are more likely to remain, even with low levels of persistence. In this scenario, revenues are generated via ads only, and as adverting effectiveness is high, ads have a good conversion rate, rendering them more profitable for the app provider. This, together with Figure 3b, is summarized by the following result:

Result 2: *When persistence decreases (satiation increases), first, the share of the paid version of the app decreases, and then with low enough persistence levels (coupled with effective levels of advertising), the app provider offers an ad-based version of the app only.*

Note that persistence in our model is correlated with retention, as it affects second-period utility, which in turn directly correlates with the decision to stay for the second period. Some anecdotal evidence supports this relationship: First, in a three-part series of notes about retention in mobile apps, Balfour (2017) listed four reasons why retention spurs apps' revenue growth, the third being that retention improves monetization. The examples given involve four monetization modes: Advertising, Subscription, Transactional, and Freemium, where in the latter, he reasons that revenues grow as a result of increased retention, as it increases upgrades from free to paid versions of apps, which is exactly the model described here.

Second, we obtained a dataset from an established publisher of multiple casual mobile game apps for children, several of which reached the Top 100 in the major app stores in recent years. Our dataset consists of data on the adoption and retention of six apps. Indeed, we observe a correlation between the percentage of users of the paid versions of the games, and retention as summarized in Table 5[10].

Table 5: Correlation between retention and % users of paid version of mobile games*

Game	% paid users	2nd week retention rate
1	2.2%	51.1%
2	0.5%	45.0%
3	2.2%	49.8%
4	1.9%	45.2%
5	2.4%	54.3%
6	1.1%	44.3%
Correlation	80.7%	

* To preserve confidentiality, all numbers are multiplied by a constant $1 \leq \theta \leq 1.3$.

6.2.1 Advertising, uncertainty, and app versions

To obtain the systematic change in the revenue share of paid vs free apps, observe Figure 3 and Result 3:

Result 3a. *With an increase in ad effectiveness, total revenues, as well as share of revenues from advertising, increase. Moreover, for low levels of ad effectiveness, most revenues are derived from the paid version of the app, while ad revenues completely dominate revenues when ad effectiveness increases.*

Result 3b. *With an increase in uncertainty, total revenues, as well as share of revenues from advertising, decrease.*

An increase in ***advertising effectiveness*** clearly increases advertising levels, and in turn the appeal of the ad version of the app. Thus, the share of revenues from the ad version increases. At very high levels of ad effectiveness, the scenario becomes pure advertising without the paid version (as per Result 2), which explains the sharp increase in the share of advertising for these high levels of k. As an increase in advertising effectiveness also increases price (as per Result 1),

[10] This cannot be construed as an empirical analysis, as we do not have data on control variables that should be controlled for in an empirical analysis. Thus this is model-free evidence for the correlation.

the app's overall profitability increases.

Figure 3a: Effect of ad effectiveness on revenues from free vs paid app versions

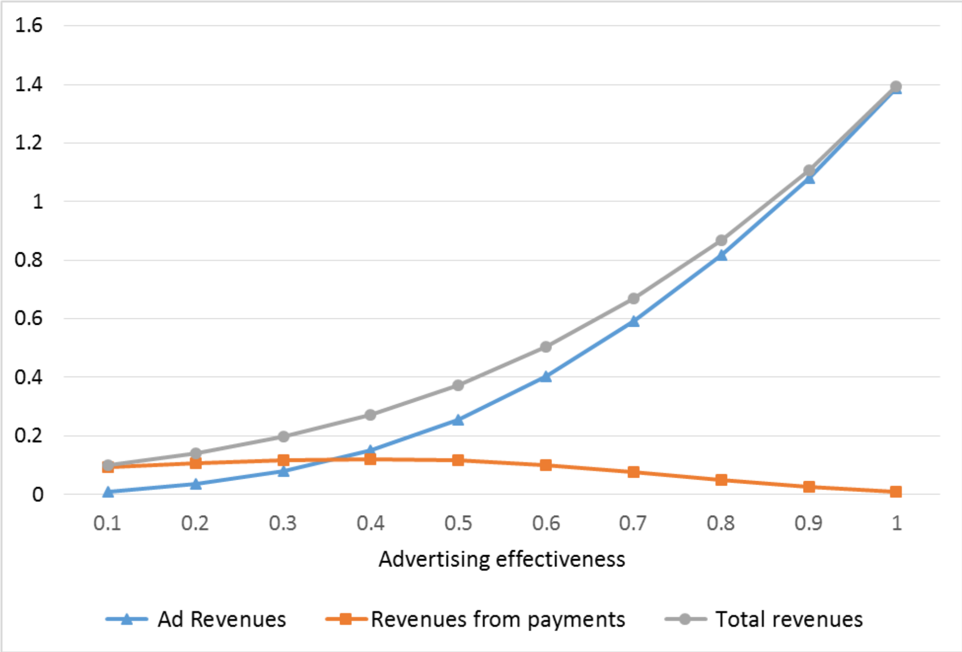
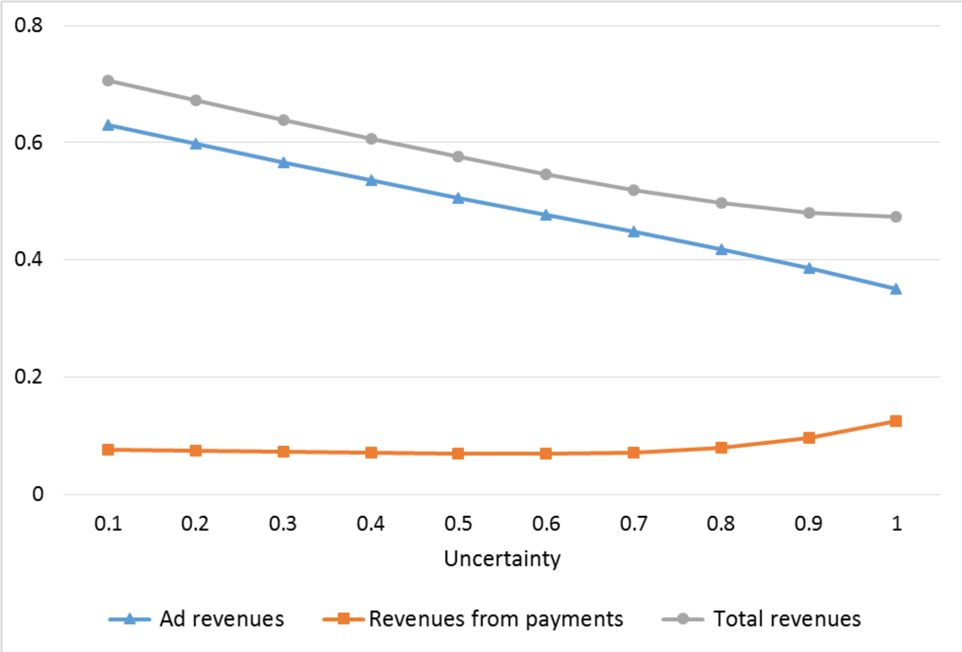


Figure 3b: Effect of uncertainty on revenues from free vs paid app version



With an increase in ***uncertainty***, we obtain two opposing effects: (1) in the pay-based scenario, price and advertising increase, and in this scenario, where uncertainty is high to begin with, most users will not use the app in the second period. Thus when uncertainty increases, there is a positive effect, as most users would not have used the app in the second period already. (2) In the ad-based scenario, advertising and price decline, and moreover in this scenario, an increase in uncertainty increases the number of those who stay only if their utility realization is positive at the expense of those who stay regardless of their utility realization. The effect is thus negative. As the proportion of ad-based scenarios is far larger than that of pay-based ones, the total effect on profitability is negative. However, this is the overall effect, and the two scenarios differ in this respect: In the ad-based scenario, uncertainty's effect on profits is negative; whereas in the pay-based scenario, Web Appendix C demonstrates that if k is smaller than δ, profits increase under uncertainty. As Table 3 shows, in the pay-based scenario, indeed k is smaller than δ. We thus checked numerically, and indeed in the two cases of the pay-based scenario, the conditions set forth in Web Appendix C hold, and thus we have the following:

Result 4. *When uncertainty increases, profits in the pay-based scenario increase, while profits in the ad-based scenario decrease.*

6.3 The role of advertising in supporting price: Damaged Good production

In a two-period model, the free version contributes to the app provider's profits not only via advertising revenues, but also by acting as sort of a "sample": Specifically, some consumers who would otherwise not use the paid app (as the paid version is too costly for them) download and use the free version to learn their fit with the app. Some of these are likely to switch in the second period to the paid version. Thus, even if advertisers pay little for ads, the app provider can benefit from offering a free version with ads. However, offering this version also has some costs: Some consumers, who would otherwise buy the paid version, might find the free version appealing. Thus, the optimal level of advertising balances between two incentives, so that the app provider's objective function depends upon ad intensity even when $k = 0$. Furthermore, *the free version with ads can yield profits even if advertisers are not paying for these ads*.

To examine this idea – that the free version with ads can yield profits even if advertisers are not paying for these ads – we focused our analysis on all cases in which $k = 0$. Indeed, in all cases wherein a paid version exists (all except for the subcase of an ad-based version with no paid version) we find that the optimal advertising level when $k = 0$ is strictly positive. The app provider has two sources of income: the consumers who downloaded the paid version in the first period; and the consumers who chose to download the free version in the first period, then switch to the paid version in the second period. The app provider does not realize any revenues from the latter segment in the first period (as $k = 0$), but does realize some profits from these consumers in the second period. This is summarized in the next statement:

Result 5. *When advertising effectiveness is zero, the app provider still advertises, despite the fact that it sees no revenues from these ads.*

This situation bears similarity to the production of damaged goods (Deneckere and McAfee 1996; Halbheer et al. 2018), where app providers deliberately "damage" an entire line of products of their offering in order to better price discriminate among their customers. An iconic example is IBM introducing the low-cost Model E laser printer that was identical to the high-cost IBM laser printer, except for the insertion of a chip that slowed printing from 10 pages per minute to 5 (Deneckere and McAfee 1996). In our case, while the app-with-ads is a damaged good, the motivation is not price discrimination: The app provider develops this app knowing that it will not obtain any direct benefit from it, as it hopes to get some consumers to download it for the sake of sampling. The app provider makes money only from the paid version of the app, despite setting an optimal level of advertising for the free version, so that the users will switch to the paid version where their utility is higher, as this version is free of advertising.

App managers can indeed use the damaged goods idea to reduce the quality of experience without compromising the quality of the content (the mobile game itself). This is particularly important as it has been shown (Li, Jain, and Kannan 2019) that quality of free version is a signal to the quality of the paid version, and damaging content of the free version may negatively impact the adoption of paid version.

7. Discussion

When looking at app markets, managers and consumers may see two broad types of apps. Some apps are by nature short term, i.e., consumers' utility may be high early on, yet soon they can expect to move to the next one. Many games and other hedonic apps fit this description. Other apps are longer-term in nature: If they fit consumers, they may stick with them for a while. Business, functional, and other utilitarian apps may better fit this category. What we showed here is that this distinction may lead to a fundamental difference in optimal management of the monetization process due to differences in propensity to stay.

We focused on the tension between the main monetization sources: paying consumers versus paying advertisers. While the tension between the monetization options is recognized as fundamental to app success (Pozin 2014), in-depth answers as to the drivers of the respective use of each are lacking. Furthermore, to the best of our knowledge, no previous studies have analyzed this issue in a customer duration dynamic setting. We show that such a setting, i.e., the combination of satiation and uncertainty, leads to some important insights on price levels, advertising intensity, profitability, and profitability sources. We summarize our findings as follows:

- We show that as satiation decreases, prices and advertising intensity increase, and the share of revenues accruing from the paid version increases. The reason is that an increase in persistence increases consumers' willingness to pay, which manifests in one of two ways: The user is willing to pay more for the app, or he is willing to be exposed to more advertising while using the app. This pushes some of those who are using the free version toward the paid version in both periods, or in Period 2. As a result, the share of consumers who use the paid version increases.
- As uncertainty increases, we see two opposing effects: (1) in the pay-based scenario where uncertainty is high to begin with, all users whose intrinsic utility is low and whose utility realization is negative, will not use the app in the second period. Thus when uncertainty increases, there is a positive effect, as most users would not have used the app in the second period in any case (with the lower uncertainty). Thus in the pay-based scenario, profits increase under uncertainty. (2) In the ad-based scenario, an increase in uncertainty increases the number of those who stay only if their utility realization is positive, at the expense of those who stay regardless of their utility realization. The effect is thus negative. As the proportion of the ad-based scenarios is far larger than the pay-based ones, the total effect on profitability is negative.
- We demonstrate that an app provider can profit from offering a free app version with ads even if advertisers are not paying for those ads. In other words, the app provider benefits from offering a version of the app that includes ads even if the consumers are not paying for the app and the advertisers are not paying for the ads. The logic behind offering this "damaged good" is that in a dynamic setting, the free version acts as a sort of "sample" that enables consumers to learn their fit with the app. The app provider would rather add ads to this version (even if they're not paid for) in order to retain the consumers who have high valuation of the app away from the "sample" and toward the paid version.

7.1 Managerial implications

The question of how to treat apps via the two main mechanisms of free (advertising) and paid (premium version, or in-app purchase) versions, is key to app management, in particular given many apps' low profitability. Thus, managers are encouraged to study the profitability outcome of the various options and understand their sources (Nathanson 2016). Our work bears a number of implications that contribute to this discussion.

Setting realistic expectations

How many paid vs free downloads should app makers expect? To what extent does the percentage of each signal an app's success? A straightforward way to look at composition of paid versus unpaid apps uses a simple segmentation scheme under which there are two kinds of users: Those who see limited value in the app, and thus will not be willing to pay; and those who see greater value and are willing to pay. Because the app maker may find it hard to assess the nature of newcomers, they will offer a free version up front, and charge some price for a minority that desires higher utility (Nathanson 2016).

While this approach helps to draw a simple-to-understand picture of the market, our analysis demonstrates that the respective shares of paid and free apps are in fact a function of a complex process. Uncertainty, satiation, and the marketing mix elements of price and advertising effectiveness combine to determine the market outcome. The advantage of a formal analytical approach lies in the ability to portray the dynamics that shape the utility function, and understand the impact of the various factors that create the emerging dynamics. This view has a direct effect when forming expectations of the profitability created by apps. For example, an ongoing debate among investors and venture capitalists surrounds the viability of freemium business models (Maltz 2012). Understanding the complexity of the situation will enable entrepreneurs and investors alike to form more educated expectations based on the app-related models; identify strengths and weaknesses; and formulate business strategies that take into account the endogenous ways in which the shares of paid versus free versions might be forecasted and realized.

Realistic expectations should also be formed regarding the effectiveness of marketing mix elements in the app monetization environment. As Result 5 suggests, advertising's effectiveness should be considered in a wider perspective that takes into account its overall effect on profitability in the free versus paid app monetization environment.

Carefully balancing investments in paid vs. free app versions

Result 2 demonstrates the delicate balance that must be struck between the free vs paid app versions in terms of investment in satiation reduction, price promotion, and advertising levels. Any change in one of these variables in turn alters the compositions of the segments that download the free vs paid app version. Furthermore, these compositions have a direct effect on the app's profitability, as Result 3 demonstrates. Moreover, the composition has an effect on retention, as paid users have higher retention rates (see Result 2 and Table 5).

An interesting issue is the effect of uncertainty: Note that the majority of users download and use free app versions, where revenues and retention, and thus lifetime value, are lower. Thus app providers look for ways to move users from free to paid app versions. Having said this, the free version plays a major role in the face of uncertainty: It is used by consumers as a sampling device, so as to better learn the app's idiosyncratic value to them. Later on they can decide to continue using the free version, or switch to the paid one, or else drop the app from further consideration. It is this role of sampling that renders uncertainty profitable under the pay-based scenario: Higher uncertainty induces users to try the free version and then switch to the paid version of the app, as in this scenario, the cost of advertising to the consumer is high (relative to price).

Investing in satiation reduction

Our results highlight in particular satiation's role, and how persistence with the app affects profitability. The contribution of customer duration to app profitability and survival is increasingly acknowledged by industry observers (Jacobson 2018). App makers are encouraged to consider user retention at the expense of acquisition, and formulate ways to deliver more value to users and be in touch with them so they will stay (Perro 2018). These efforts are particularly notable for mobile apps given the difficulty in retaining adopters of free products (Datta, Foubert, and Van Heerde 2015). While there can be large variation among apps, using new methods managers should have increased ability to can forecast app usage and predict the number of ads that will be shown, based on past users' data (Rutz, Aravindakshan, and Rubel 2019).

Given the expected effect of satiation on app use duration, as is also cited here, app creators can draw from the emerging behavioral literature on satiation. For example, to decrease satiation, managers may want to encourage users to change their consumption rates (Galak, Kruger, and Loewenstein 2013), or to imagine future variety (Sevilla, Zhang, and Kahn 2016). Apps may want to manage the similarity of new experiences that users obtain (Lasaleta and Redden 2018), or limit the times for which the products are available (Sevilla and Redden 2014).

Yet the road from persistence to retention is not always linear and straightforward. As Result 2 shows, satiation is correlated with the share of users who download the free version of the app, which in turn affects both profitability from current users and retention, as free users have lower retention rates. What we see is the indirect way in which satiation affects the marketing mix. The classic effect of retention on profit is considered to be direct, through customer lifetime value (Ascarza et al. 2018). Persistence will certainly have an effect in this direction. Yet as we see in Result 1, persistence also has a positive impact on advertising levels and on the price to be charged for the paid version. App creators should thus intensively examine ways to decrease satiation, and consider carefully the positive consequences of these efforts.

7.2 Limitations

Our attempt to capture a complex market situation with a relatively parsimonious model clearly has a number of limitations. One issue is the lack of explicit modeling of competitive activity, which takes into account alternative space, time, and competitive apps. The added complexity thereof is beyond our scope here. Moreover, it is not trivial to model explicit competition, due to the complex meaning of what constitutes a “competitor” in this market. Many free products compete in a general sense for users’ attention and time, and not necessarily with products to which they are very similar. However, there may be specific categories in which clear, direct competitors emerge.

We examined a specific freemium model in which the product’s premium version had no advertising. There are more complex free business models in which, for example, users purchase in-app add-ons that change the product’s utility in various ways. Modeling each of these scenarios would add complexity that we feel is unneeded at this stage, yet can of course be done in future explorations. In the same vein, we assumed a fixed price for the paid version. This assumption is supported by market observations and interviews with managers, which suggest that given the short time periods, the paid version price is largely fixed. Changing this assumption adds significant complexity, yet can be an interesting avenue for future research.

However, one growing monetization strategy in in app purchases is rewarded ads, where users watch an ad instead of paying for an in-app purchase (Wiggers 2019). In a recent survey by Google/Kantar, 64% of players who do not pay for games were willing to watch a rewarded ad instead (Google 2019). This new strategy can be nested in our model, where consumers can watch ads in exchange for more content, or pay for that content.

Finally, further analysis might focus on peer effects, which are absent from our analysis. When a customer leaves, it may affect not only the app provider's ability to acquire other customers via word of mouth, but may also lead to the churn of other current customers (Nitzan and Libai 2011). This may further underscore the importance of efforts to decrease satiation, and increase customer utility in general. Furthermore, existing paying customers may affect other customers' utility by creating within-customer contagion processes (Bapna and Umyarov 2015). In this sense, there is still a way to go toward better understanding of the complex process in which profitability is created when monetizing free versus paid apps.

References

Ackerberg, D. 2003. Advertising, learning, and consumer choice in experience good markets: An empirical examination. *Internat. Econom. Rev.* **44**(3), 1007-1040.

AppBrain. 2019. Android and Google Play statistics.

Arora, S., F. T. Hofstede, and V. Mahajan. 2017. The implications of offering free versions for the performance of paid mobile apps. *J. of Marketing* **81**(3), 62-78.

Ascarza, E., S. Neslin, O. Netzer, Z. Anderson, P. Fader, S. Gupta, B. Hardie, A. Lemmens, B. Libai, D. Neal, F. Provost, and R. Schrift. 2018. In pursuit of enhanced customer retention management. *Customer Need and Solutions* **5**(1-2), 65-81.

Assmus, G., J. Farley, and D. Lehmann. 1984. How advertising affects sales: Meta-analysis of econometric results. *J. Marketing Res*. **21**(1), 65-74.

Balfour, B. 2017. Growth loops are the new funnels. *Reforge*.

Babu, S. 2018. Are you mobile ready? The state of mobile conversions report will tell you. Martech Advisor, January 10.

Bapna, R. and A. Umyarov. 2015. Do your online friends make you pay? A randomized field experiment on peer influence in online social networks. *Management Sci*. **61**(8), 1902-1920.

Biyalogorsky, E. and O. Koenigsberg. 2014. The design and introduction of product lines when consumer valuations are uncertain. *Production and Operations Management* **23**(9), 1539-1548.

Cheney, S. and E. Thompson. 2018. The 2017-2022 app economy forecast: 6 billion devices, $157 billion in spend and more. *App Annie*.

Datta, H., B. Foubert, and H. J. Van Heerde. 2015. The challenge of retaining customers acquired with free trials. *J. Marketing Res.* **52**(2), 217-234.

Deneckere, R. J. and Preston McAfee, R. 1996. Damaged goods. *Journal of Economics & Management Strategy* **5**(2), 149-174.

Deng, Y., A. Lambrecht, and Y. Liu. 2018. Spillover effects and freemium strategy in mobile app market. *SSRN.*

Erdem, T., J. Swait, S. Broniarczyk, D. Chakravarti, J. Kapferer, M. Keane, J. Roberts, J. B. Steenkamp, and F. Zettelmeyer. 1999. Brand equity, consumer learning, and choice. *Marketing Lett.* **10**(3), 301-318.

Freier, A. (2018); "App revenue reaches $92.1 billion in 2018 driven by mobile gaming apps". Business of Apps, September 13.

Foubert, B. and E. Gijsbrechts. 2016. Try it, you'll like it—or will you? The perils of early free-trial promotions for high-tech service adoption. *Marketing Sci.* **35**(5), 810-826.

Galak, J., J. Kruger, and G. Loewenstein. 2013. Slow down! Insensitivity to rate of consumption leads to avoidable satiation. *J of Cons. Research* **39**(5), 993-1009.

Galak, J. and J. P. Redden. 2018. The properties and antecedents of hedonic decline. *Ann Rev Psych*. 69, 1-25.

Gu, X., P. K. Kannan, and L. Ma. 2018. Selling the premium in freemium. *J of Marketing* **82**(6), 10-27.

Godes, D., E. Ofek, and M. Sarvary. 2009. Content vs. advertising: The impact of competition on media firm strategy. *Marketing Sci.* **28**(1), 20-35.

Goldstein, D. G., S. Suri, R. P. McAfee, M. Ekstrand-Abueg, and F. Diaz. 2014. The economic and cognitive costs of annoying display advertisements. *J. Marketing Res.* **51**(6), 742-752.

Google. 2019. Why mobile gaming is the future of leisure time. *Think with Google*, April.

Halbheer, D., D. L. Gärtner, E. Gerstner, and O. Koenigsberg. 2018. Optimizing service failure and damage control. *Internat. J. Res Marketing* **35**(1), 100-115.

Halbheer, D., F. Stahl, O. Koenigsberg, and D. R. Lehmann. 2014. Choosing a digital content strategy: How much should be free? *Internat. J. Res. Marketing* **31**(2), 192-206.

Han, S. P., S. Park, and W. Oh. 2016. Mobile app analytics: A multiple discrete-continuous choice framework. *MIS Quarterly* **40**(4), 983-1008.

Haslam, J. 2018. Mobile Gaming Benchmarks Q1 2018. *Adjust*, June 25.

Hong, Y. and P. A. Pavlou. 2014. Product fit uncertainty in online markets: Nature, effects, and antecedents. *Inform. Systems Res.* **25**(2), 328-344.

Hui, S. K. (2017). Understanding repeat playing behavior in casual games using a Bayesian data augmentation approach. *Quant. Marketing and Econo*. **15**(1), 29-55.

Hsu, C-L. and J. C-C. Lin 2016. Effect of perceived value and social influences on mobile app stickiness and in-app purchase intention. *Tech. Forecasting Soc. Change* **108**, 42-53.

Jacobson, B. 2018. How mobile app retention is evolving. *PostFunnel*, July 19.

Iyengar, R., A. Ansari, and S. Gupta. 2007. A model of consumer learning for service quality and usage. *J. Marketing Res.* **44**(3), 529-544.

Kannan, P. K. 2013. Designing and pricing digital content products and services: A research review. *Rev. Marketing Res.* **10**, 97-114.

Kannan, P. K. and H. A. Li. 2017. Digital marketing: A framework, review, and research agenda. *Internat. J. Res. Marketing* **34**(1), 22-45.

Klotzbach, C. 2016. Enter the matrix: App retention and engagement. *Flurry Insights*, May 12.

Lagace, M. 2018. Mobile gaming has an advertising problem that needs to be addressed. *AndroidCentral*, Feb 21.

Lambrecht, A., A. Goldfarb, A. Bonatti, A. Ghose, D. Goldstein, R. Lewis, A. Rao, N. Sahni, and S. Yao. 2014. How do firms make money selling digital goods online? *Marketing Lett.* **25**(3), 331-341.

Lambrecht, A. and K. Misra. 2016. Fee, or free? When should firms charge for online content? *Management Sci.* **63**(4), 1150-1165.

Lasaleta, J. D. and Redden, J. P. 2018. When promoting similarity slows satiation: The relationship of variety, categorization, similarity, and satiation. *J. of Marketing Res.* **55**(3), 446-457.

Lee, C., V. Kumar, and S. Gupta. 2017. Designing freemium: Balancing growth and monetization strategies. SSRN.

Li, H. A., S. Jain, and P. K. Kannan. 2019. Optimal design of content samples for digital products and services. *J. Marketing Res*. forthcoming.

Libai, B., E. Muller, and R. Peres. 2009. The diffusion of services. *J. Marketing Res.* **46**(2), 163-175.

Maltz, J. 2012. Should your startup go freemium? *TechCrunch*.

Moorthy, K. S., 1984. Market segmentation, self-selection, and product line design. *Marketing Sci.* **3**(4), 288-307.

Moorthy, K. S. and Png, I. P. 1992. Market segmentation, cannibalization, and the timing of product introductions. *Management Sci.* **38**(3), 345-359.

Morvinski, C., O. Amir, and E. Muller. 2017. "Ten million readers can't be wrong!" Or can they? On the role of information about adoption stock in new product trial. *Marketing Sci.* **36**(2), 290-300.

Musalem, A. and Y. V. Joshi. 2009. How much should you invest in each customer relationship? A competitive strategic approach. *Marketing Sci.* **28**(3), 555-565.

Natanson, E. 2016. "Ebony and ivory": The blended in-app purchase and advertising model. *Forbes*, July 7.

Needleman, S. E. 2016. How mobile games rake in billions. *Wall Street Journal*, July 28.

Niculescu, M. F. and D. J. Wu. 2014. Economics of free under perpetual licensing: Implications for the software industry. *Informat. Systems Res.* **25**(1), 173-199.

Ogg, E. 2013. Why even the best iOS app developers struggle to set the right price. *Gigaop*, June 19.

Pauwels, K. and A. Weiss. 2008. Moving from free to fee: How online firms market to change their business model successfully. *J. Marketing* **72**(3), 14-31.

Perro, J. (2018). Mobile apps: What's a good retention rate? *Localytics*, March 22.

Pindyck, R. S. 1982. Adjustment costs, uncertainty, and the behavior of the firm. *American Economic Review* **72**(3), 415-427.

Pozin, I. 2014. How to monetize your app: Banner ads vs native ads vs in-app purchases. *Forbes*, December 31.

Prasad, A., V. Mahajan, and B. Bronnenberg. 2003. Advertising versus pay-per-view in electronic media. *Internat. J. Res. Marketing* **20**(1), 13-30.

Rutz, O., Aravindakshan, A. and Rubel, O., 2019. Measuring and forecasting mobile game app engagement. *Internat. J. Res. Marketing* **36**(2), 185-199.

Schulze, C., L. Schöler, and B. Skiera. 2014. Not all fun and games: Viral marketing for utilitarian products. *J of Marketing* **78**(1), 1-19.

Sevilla, J. and J. P. Redden. 2014. Limited availability reduces the rate of satiation. *J. Marketing Res*. **51**(2), 205-217.

Sevilla, J., J. Zhang, and B. E. Kahn. 2016. Anticipation of future variety reduces satiation from current experiences. *J. Marketing Res.* **53**(6), 954-968.

Shin, J. and K. Sudhir. 2012. When to "fire" customers: Customer cost-based pricing. *Management Sci.* **58**(5), 932-947.

Statista. 2018. Number of apps available in leading app stores as of 1st quarter 2018.

Subramanian, U., J. S. Raju, and J. Zhang. 2013. The strategic value of high-cost customers. *Management Sci.* **60**(2), 494-507.

Tåg, J. 2009. Paying to remove advertisements. *Informat. Economics Policy* **21**(4), 245-252.

Vakratsas, D. and Ambler, T. 1999. How advertising works: What do we really know? *Journal of Marketing* **63**(1), 26-43.

Wiggers, K. 2019. Google is giving users in-app rewards for watching ads. *VB*, March 6.

Wijman, T. 2018. Mobile revenues account for more than 50% of the global games market as it reaches $137.9 billion in 2018. *Newzoo* April 30.

Wilbur, K. C. 2008. A two-sided, empirical model of television advertising and viewing markets. *Marketing Sci.* **27**(3), 356-378.

Appendices

Appendix A: Consumers' choices of free vs paid app versions

In this appendix, we consider the two options that the user faces when she adopts the app. She can either choose the paid version with U_p utility, or she can choose the free option, with utility U_a. While U_p is straightforward, U_a describes two distinct cases, both of which are equivalent for low and high values of α. For low values of α, only users with positive utility realization will use the free app in the second period; and for high values of α, users with negative realization will use the free app in the second period; while users with positive utility realization will switch to the paid app in the second period. The difference between the cases lies in intermediate levels of α depending on two thresholds for α: The first is ε, the threshold for users with negative realization for adopting the free app. The second is $p/\delta\gamma - \varepsilon$, the threshold for free users with positive realization to switching from the free to the paid app. Thus we have two cases depending upon the relative values of these two thresholds. Note that the condition $\varepsilon < p/\gamma\delta - \varepsilon$ is satisfied if (and only if) $\varepsilon < p/2\gamma\delta$.

<u>Case 1</u>: $\varepsilon < p/2\gamma\delta$

In this case, the slopes of the utility segments of U_a are given by the following equations:

$$\text{(A1)}\ \frac{\partial U_a}{\partial \alpha} = \begin{Bmatrix} (1-\gamma)\left(1+\frac{\delta}{2}\right) \text{for } \alpha < \varepsilon \\ (1-\gamma)(1+\delta) \text{ for } \varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon \\ (1-\gamma)\left(1+\frac{\delta}{2}\right) + \frac{\delta}{2} \text{ for } \frac{p}{\gamma\delta} - \varepsilon < \alpha \end{Bmatrix}$$

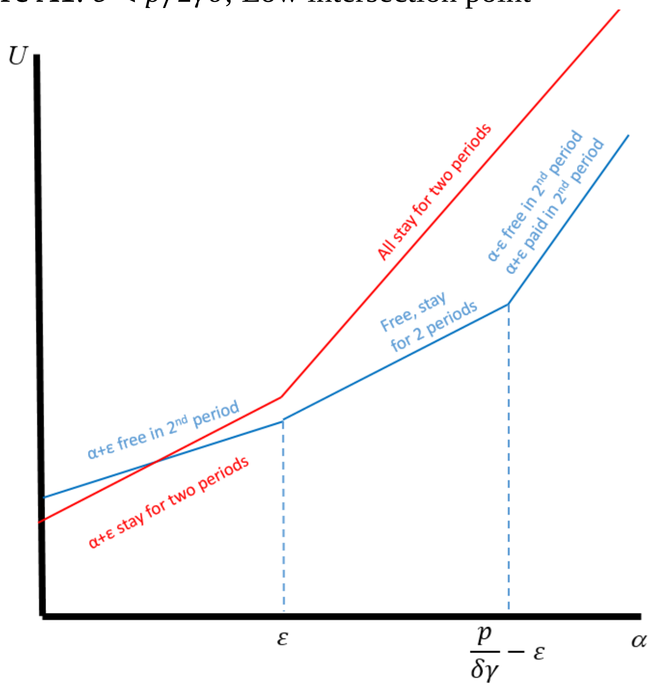
While the slope of U_p is:

$$\text{(A2)}\ \frac{\partial U_p}{\partial \alpha} = \begin{Bmatrix} 1+\frac{\delta}{2} \quad \text{for} \quad \alpha < \varepsilon \\ 1+\delta \quad \text{for} \quad \varepsilon < \alpha \end{Bmatrix}$$

Note that at the intersection point of U_p and U_a (i.e., for the same level of α), the slope of U_p is steeper than that of U_a. It is also straightforward to see that when $\alpha = 0$, $U_a(0) > U_p(0)$ for all cases of Appendix B (except for the last case that we prove cannot exist). This will be evident in all subsequent figures in both appendices. As the slope of U_p makes a discrete change at $\alpha = \varepsilon$, and this corresponds to the exact same point in the change of slope of U_a, the utilities can have three possible intersection locations as follows:

Case 1.1: $\varepsilon < p/2\gamma\delta$; Low intersection point: The intersection occurs at $\alpha < \varepsilon$. Once this intersection occurs, the lines cannot intersect again, as U_p will always be greater than U_a, as its slope is greater. See Equations A1 and A2, and Figure A1.

Figure A1: $\varepsilon < p/2\gamma\delta$; Low intersection point



At the point of intersection,

$$U_p = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon) = \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon)\right] \cdot (1 - \gamma) = U_a$$

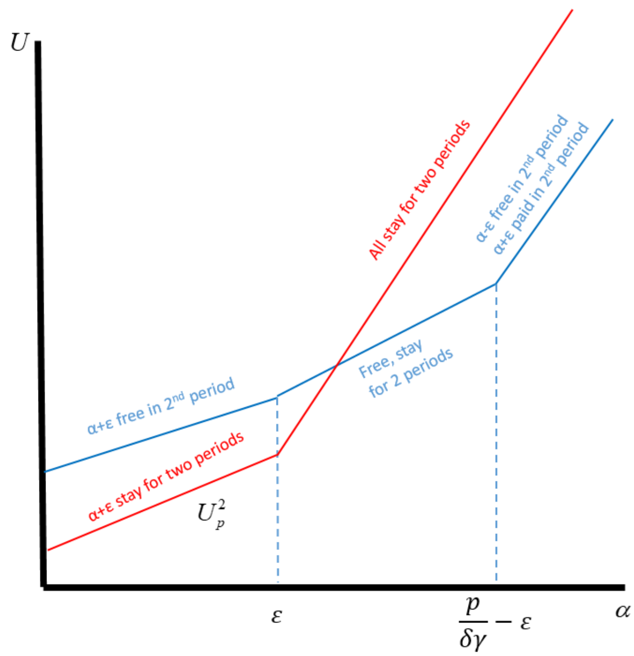
$p = \gamma\left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon)\right]$; or $\alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2+\delta)}$

As in this case, by definition, $\varepsilon < \frac{p}{\delta\gamma} - \varepsilon$ the lines intersect when

$\gamma\left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon)\right] > 2\varepsilon\delta\gamma$, or when $\alpha > \frac{3\varepsilon\delta}{2+\delta}$. Furthermore, as $\alpha < \varepsilon$, the lines intersect when $\frac{3\varepsilon\delta}{2+\delta} < \alpha < \varepsilon$. Replacing α with $\frac{2p - \gamma\varepsilon\delta}{\gamma(2+\delta)}$ defines the range of ε for this case, where $\frac{3\varepsilon\delta}{2+\delta} < \frac{2p - \gamma\varepsilon\delta}{\gamma(2+\delta)} < \varepsilon$; or $\frac{p}{\gamma(1+\delta)} < \varepsilon < \frac{p}{2\delta\gamma}$.

Case 1.2: $\varepsilon < p/2\gamma\delta$; Medium intersection point: The intersection occurs at $\varepsilon < \alpha < \frac{p}{\gamma\delta} - \varepsilon$. Once this intersection occurs, the lines cannot intersect again, as U_p will remain above U_a, as its slope is greater. See Equations A1 and A2, and Figure A2.

Figure A2: $\varepsilon < p/2\gamma\delta$; Medium intersection point
At the point of intersection,



$U_a = \alpha(1-\gamma)(1+\delta) = \alpha - p + \alpha\delta = U_p$

$p = \alpha\gamma(1+\delta)$; or $\alpha = \frac{p}{\gamma(1+\delta)}$

However, as the intersection is greater than ε, we also require $\alpha = \frac{p}{\gamma(1+\delta)} > \varepsilon$, defining the range of ε for this case as well.

Case 1.3: $\varepsilon < p/2\gamma\delta$; High intersection point: The intersection occurs at $\alpha > \frac{p}{\gamma\delta} - \varepsilon$. As we show here, this case does not exist, i.e., the conditions under which it exists violate the requirement of Case 1. Assume, *a contrario*, that the intersection does occur, and thus at the point of intersection,

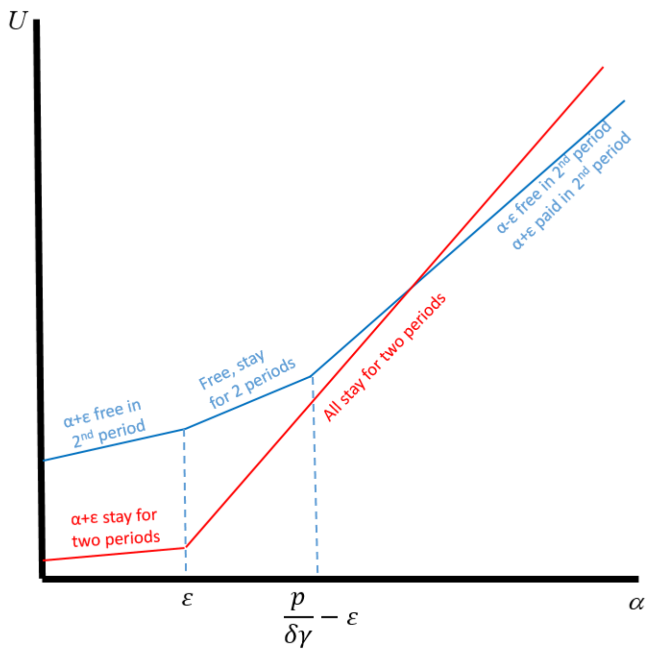
$$U_a = \alpha(1-\gamma) + \frac{\delta}{2}(\alpha - \varepsilon)\cdot(1-\gamma) + \frac{\delta}{2}(\alpha+\varepsilon) - \frac{p}{2} = \alpha - p + \alpha\delta = U_p$$

$$\alpha(1-\gamma) + \frac{\delta}{2}(\alpha - \varepsilon)\cdot(1-\gamma) + \frac{\delta}{2}(\alpha+\varepsilon) + \frac{p}{2} - \alpha(1+\delta) =$$

$$= \alpha - \alpha\gamma + \left(\frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2}\right)(1-\gamma) + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} =$$

$$= \alpha - \alpha\gamma + \frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2} - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} = -\alpha\gamma - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{p}{2} = 0$$

Figure A3: $\varepsilon < p/2\gamma\delta$; High intersection point



$p = \gamma[2\alpha + \delta(\alpha - \varepsilon)]$; or $\alpha = \frac{p+\gamma\varepsilon\delta}{\gamma(2+\delta)}$

Here $\alpha > \frac{p}{\gamma\delta} - \varepsilon$, so that the lines intersect when $\frac{p+\gamma\varepsilon\delta}{\gamma(2+\delta)} > \frac{p}{\gamma\delta} - \varepsilon$ or when $p < \gamma\varepsilon\delta(1+\delta)$;

However, in this case, $\varepsilon < \frac{p}{\gamma\delta} - \varepsilon$, that is, $p > 2\varepsilon\delta\gamma > \gamma\varepsilon\delta(1+\delta)$, in contrast to the above condition on p. Thus the utility lines do not intersect, ergo Case 1.3 does not exist.

Case 2: $\varepsilon > p/2\gamma\delta$

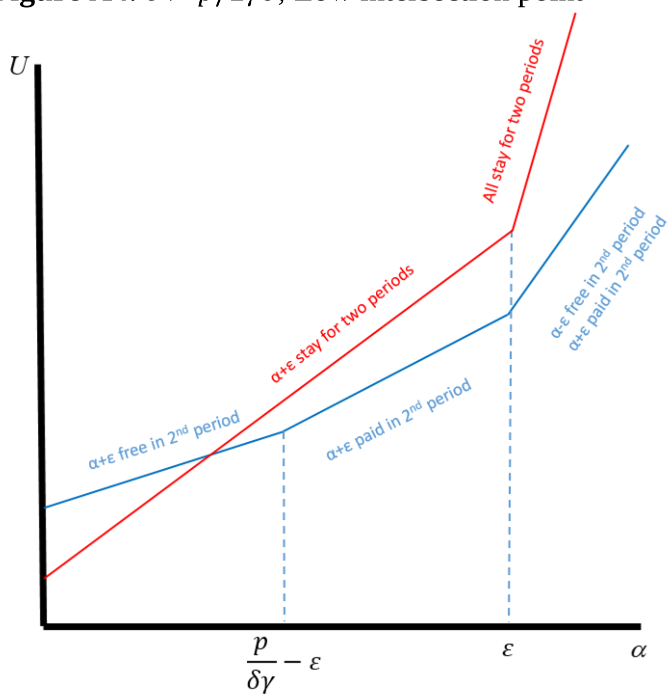
In this case, the slopes of the utility segments of U_a are given by the following equations:

$$\text{(A3)}\ \frac{\partial U_a}{\partial \alpha} = \begin{cases} (1-\gamma)\left(1+\frac{\delta}{2}\right) & \text{for } \alpha < \frac{p}{\gamma\delta} - \varepsilon \\ 1-\gamma+\frac{\delta}{2} & \text{for } \frac{p}{\gamma\delta} - \varepsilon < \alpha < \varepsilon \\ (1-\gamma)\left(1+\frac{\delta}{2}\right)+\frac{\delta}{2} & \text{for } \varepsilon < \alpha \end{cases}$$

while U_p's slope is the same as in Equation (A2). As in Case 1, U_a starts at a higher value than U_p, and at the intersection point of U_p and U_a, U_p's slope is greater than that of U_a, and as the slope of U_p changes at $\alpha = \varepsilon$, and this corresponds to the exact same point in the change of U_a's slope, the utilities can have three possible intersection locations as follows:

Case 2.1: $\varepsilon > p/2\gamma\delta$; Low intersection point: The intersection occurs at $\alpha < \frac{p}{\gamma\delta} - \varepsilon$. Once this intersection occurs, the lines cannot intersect again, as U_p will remain above U_a, as its slope is greater. See Equations A2 and A3, and Figure A4.

Figure A4: $\varepsilon > p/2\gamma\delta$; Low intersection point



At the point of intersection,

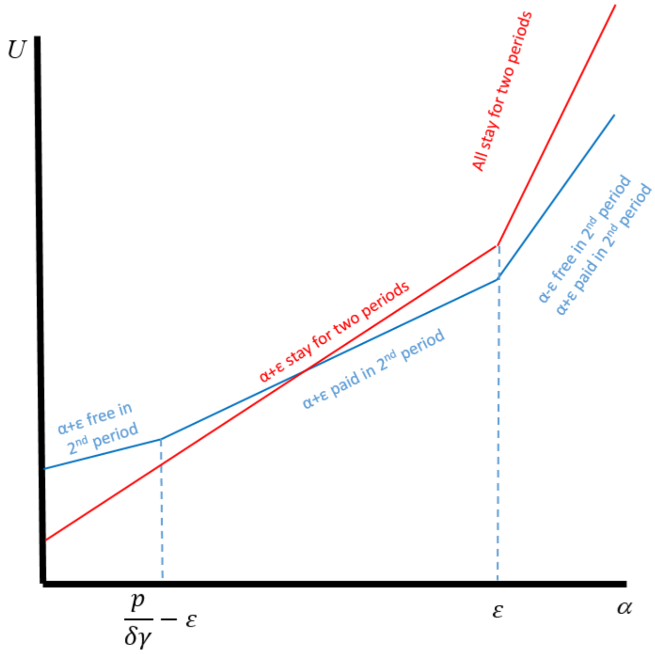
$$U_p = \alpha - p + \frac{\delta}{2}(\alpha + \varepsilon) = \left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon)\right] \cdot (1 - \gamma) = U_a$$

$p = \gamma\left[\alpha + \frac{\delta}{2}(\alpha + \varepsilon)\right]$; or $\alpha = \frac{2p - \gamma\varepsilon\delta}{\gamma(2+\delta)}$

We still need to ascertain that $\alpha = \frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)} < \frac{p}{\gamma\delta} - \varepsilon$, or that $\varepsilon < \frac{p(2-\delta)}{2\gamma\delta}$, including the case limit on ε that defines ε's range in this case: $\frac{p}{2\gamma\delta} < \varepsilon < \frac{p(2-\delta)}{2\gamma\delta}$

Case 2.2: $\varepsilon > p/2\gamma\delta$; Medium intersection point: The intersection occurs at $\frac{p}{\gamma\delta} - \varepsilon < \alpha < \varepsilon$. Once this intersection occurs, the lines cannot intersect again, as U_p will always be greater than U_a, as its slope is greater. See Equations A2 and A3, and Figure A5.

Figure A5: $\varepsilon > p/2\gamma\delta$; Medium intersection point



$$U_a = \alpha(1-\gamma) + \frac{\delta}{2}(\alpha+\varepsilon) - \frac{p}{2} = \alpha - p + \frac{\delta}{2}(\alpha+\varepsilon) = U_p$$

$p = 2\alpha\gamma$; or $\alpha = \frac{p}{2\gamma}$

In this case, $\varepsilon > \frac{p}{2\gamma\delta}$, that is, $p < 2\varepsilon\delta\gamma$, so that the lines intersect when $2\alpha\gamma < 2\varepsilon\delta\gamma$ or $\alpha < \varepsilon\delta$.

Replacing α with $\frac{p}{2\gamma}$ defines ε's range for this case as $\varepsilon > \frac{p}{2\delta\gamma}$.

Case 2.3: $\varepsilon > p/2\gamma\delta$; High intersection point: The intersection occurs at $\alpha > \varepsilon$. As we show here, this case does not exist, that is, the conditions under which it exists violate the requirement of Case 2. Assume, *a contrario*, that the intersection does occur (see Figure A6), and thus at the point of intersection,

$$U_a = \alpha(1-\gamma) + \frac{\delta}{2}(\alpha-\varepsilon)\cdot(1-\gamma) + \frac{\delta}{2}(\alpha+\varepsilon) - \frac{p}{2} = \alpha - p + \alpha\delta = U_p$$

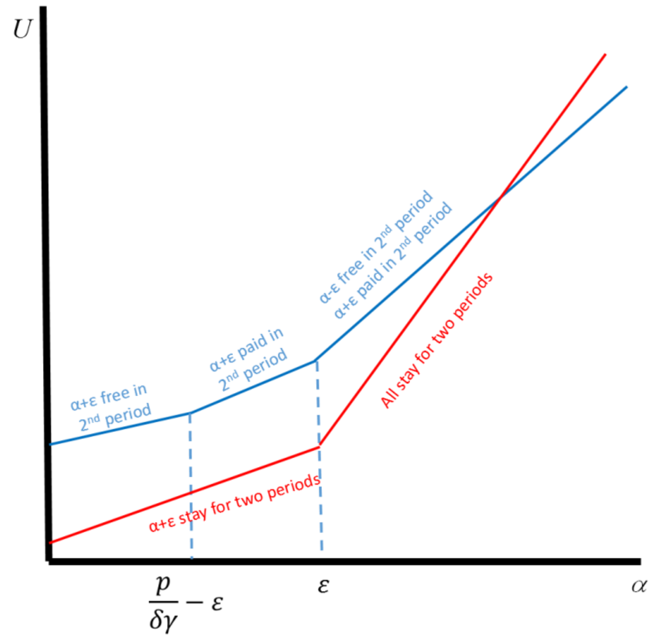
$$\alpha(1-\gamma) + \frac{\delta}{2}(\alpha-\varepsilon)\cdot(1-\gamma) + \frac{\delta}{2}(\alpha+\varepsilon) + \frac{p}{2} - \alpha(1+\delta) =$$

$$= \alpha - \alpha\gamma + \left(\frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2}\right)(1-\gamma) + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} =$$

$$= \alpha - \alpha\gamma + \frac{\delta\alpha}{2} - \frac{\delta\varepsilon}{2} - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{\delta\alpha}{2} + \frac{\delta\varepsilon}{2} - \alpha - \alpha\delta + \frac{p}{2} =$$

$$= -\alpha\gamma - \frac{\delta\gamma\alpha}{2} + \frac{\delta\gamma\varepsilon}{2} + \frac{p}{2} = 0$$

Figure A6: $\varepsilon > p/2\gamma\delta$; High intersection point



$p = \gamma[2\alpha + \delta(\alpha - \varepsilon)]$; or $\alpha = \frac{p+\gamma\varepsilon\delta}{\gamma(2+\delta)}$.

In this case, $\varepsilon > \frac{p}{2\gamma\delta}$, or $p < 2\varepsilon\delta\gamma$, so that the lines intersect when $p = \gamma[2\alpha + \delta(\alpha - \varepsilon)] < 2\varepsilon\delta\gamma$ or when $\alpha < \frac{3\varepsilon\delta}{2+\delta}$. However, $\frac{3\varepsilon\delta}{2+\delta} < \varepsilon$, contradicting the fact that the intersection occurs at a point that is greater than ε. Thus Case 2.3 cannot happen.

Summary of Case 1: When $\varepsilon < \frac{p}{2\gamma\delta}$, U_p's and U_a's utilities can only intersect in Cases 1.1 and 1.2. Case 1.1's range is when $\frac{p}{\gamma(1+\delta)} < \varepsilon < \frac{p}{2\delta\gamma}$, and then $\alpha = \frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)}$, and Case 1.2's range is when $\varepsilon < \frac{p}{\gamma(1+\delta)}$, and then $\alpha = \frac{p}{\gamma(1+\delta)}$.

Summary of Case 2: When $\varepsilon > \frac{p}{2\gamma\delta}$, U_p's and U_a's utilities can only intersect in Cases 2.1 and 2.2. Case 2.1's range is when $\frac{p}{2\gamma\delta} < \varepsilon < \frac{p(2-\delta)}{2\gamma\delta}$, and then $\alpha = \frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)}$, and Case 2.2's range is when $\varepsilon > \frac{p}{2\delta\gamma}$, and then $\alpha = \frac{p}{2\gamma}$.

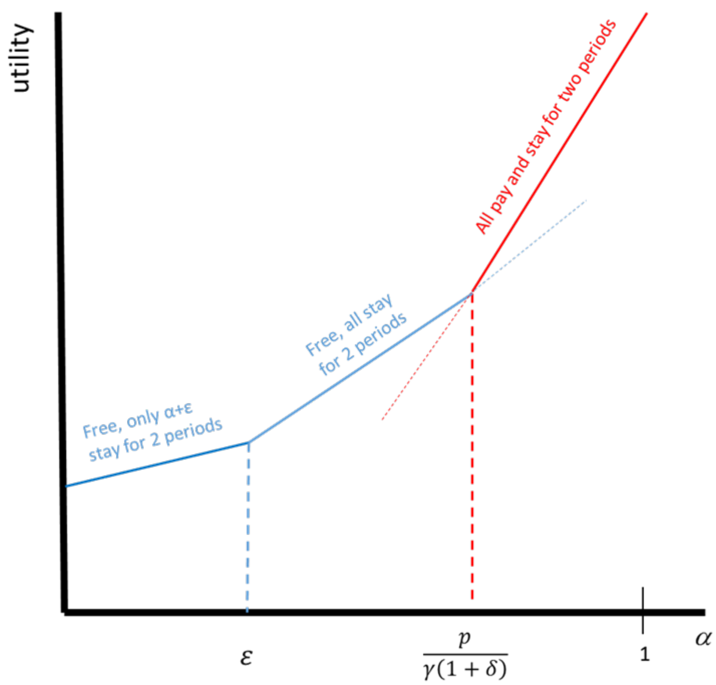
Appendix B: The firm's profit function

While in Appendix A we considered the two options that the user faces when he adopts the app, and found the four cases specified at the end of the appendix; in this appendix, we group the four cases into the two scenarios presented in the paper: Ad-Based, and Pay-Based, each with its own special case as described shortly. We begin with the Ad-Based Scenario, that is, Case 1.2 in Appendix A, and its special case. We then proceed to the Pay- Based Scenario – Case 2.2 of Appendix A – and its special case (Cases 1.1 and 2.1). We conclude by showing that a second special case of the Pay-Based Scenario cannot exist, as it violates one of the conditions of the model's parameters.

Ad-Based Scenario: Case 1.2 of Appendix A

Observe the following figure (Figure 1 in the text):

Figure B1: Ad-Based Scenario



In this case, when $\alpha < \varepsilon$, everyone uses the free version in the first period, and those with positive utility realization stay for a second period. When $0 < \alpha < \frac{p}{\gamma(1+\delta)}$, everyone uses the free version in both periods. Finally, when $> \frac{p}{\gamma(1+\delta)}$, everyone uses the paid version for both periods. We use the firm's objective function as defined in Equation (1), with one addition: We define P_{12} as the share of those who paid in Period 1 and stayed in Period 2: While they do not generate any profit to the firm, and thus do not appear in Equation (1), we report their value here for completeness' sake.

Here, $P_1 = P_{12} = 1 - \frac{p}{\gamma(1+\delta)}$; $P_2 = 0$; $A_1 = \frac{p}{\gamma(1+\delta)}$; and $A_2 = \frac{2p-\gamma\varepsilon(1+\delta)}{2\gamma(1+\delta)}$

$$\Pi = p\left(1 - \frac{p}{\gamma(1+\delta)}\right) + k\gamma\frac{4p - \gamma\varepsilon(1+\delta)}{2\gamma(1+\delta)} - \gamma^2$$

Differentiating with respect to *p*, equating to zero, and collecting terms yield:

$$p_{opt} = \frac{\gamma}{2}(1 + 2k + \delta)$$

to be in this scenario requires $\gamma(1+\delta) > \frac{\gamma}{2}(1 + 2k + \delta) > \varepsilon\gamma(1+\delta)$,

or: $\left(\varepsilon - \frac{1}{2}\right)(1+\delta) < k < \frac{1+\delta}{2}$

Differentiating with respect to *γ*, equating to zero, and collecting terms yield:

$$\frac{p^2}{\gamma^2(1+\delta)} - \frac{k\varepsilon}{2} - 2\gamma = 0$$

Substitute the optimal *p* from the above equation, and solve to get the optimal *γ*:

$$\gamma_{opt} = \frac{(1 + 2k + \delta)^2 - 2k\varepsilon(1+\delta)}{8(1+\delta)}$$

Now substitute γ_{opt} into p_{opt} to obtain the solution of optimal price as a function of the parameters.

$$p_{opt} = (1 + 2k + \delta)\frac{(1 + 2k + \delta)^2 - 2k\varepsilon(1+\delta)}{16(1+\delta)}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{[(1 + 2k + \delta)^2 - 2k(1+\delta)\varepsilon]^2}{64(1+\delta)^2}$$

The partial derivatives of the optimal advertising, price, and profit levels are:

$$\frac{\partial\gamma}{\partial\varepsilon} = -\frac{k}{4} < 0$$

$\frac{\partial\gamma}{\partial\delta} = \frac{(1+\delta)^2 - 4k^2}{8(1+\delta)^2} \geq 0$, as $\frac{1+\delta}{2} > k$

$$\frac{\partial\gamma}{\partial k} = \frac{2-\varepsilon}{4} + \frac{k}{(1+\delta)} > 0$$

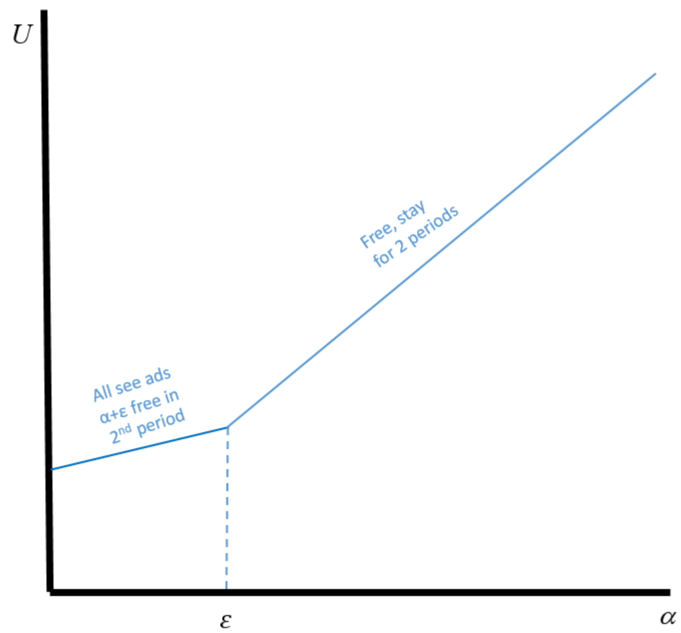
$$\frac{\partial p}{\partial\varepsilon} = -\frac{k(1 + 2k + \delta)}{8} < 0$$

$\frac{\partial p}{\partial \delta} = \frac{(1+\delta)^3 - 4k^3 + k(1+\delta)^2(3-\varepsilon)}{8(1+\delta)^2} \geq 0$, as $\frac{1+\delta}{2} > k, \varepsilon < 1$

$$\frac{\partial p}{\partial k} = \frac{3(1+2k+\delta)^2 - \varepsilon(1+\delta)(1+4k+\delta)}{8(1+\delta)} > 0$$

Ad-Based Scenario, special case: For Figure B1 to hold, we need $\frac{p}{\gamma(1+\delta)} < 1$ to be true. In this special case, we address the situation wherein the reverse holds, that is, $\frac{p}{\gamma(1+\delta)} \geq 1$. Thus the two lines (paid and free) do not intersect, as the price is relatively high compared to the level of advertising, so that no one pays for the app. See Figure B2.

Figure B2: Ad-Based Scenario, special case



In this case, when $\alpha < \varepsilon$, everyone uses the free version in Period 1, and those with positive utility realization stay for a second period. When $\alpha > \varepsilon$, they stay for both periods.

Here, $P_1 = P_{12} = 0$; $P_2 = 0$; $A_1 = 1$; and $A_2 = 1 - \varepsilon + \frac{\varepsilon}{2}$

$$\Pi = k\gamma\left(2 - \frac{\varepsilon}{2}\right) - \gamma^2$$

Differentiating with respect to γ, equating to zero, and collecting terms yield:

$$\gamma_{opt} = \frac{k(4-\varepsilon)}{4}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{k^2(-4+\varepsilon)^2}{16}$$

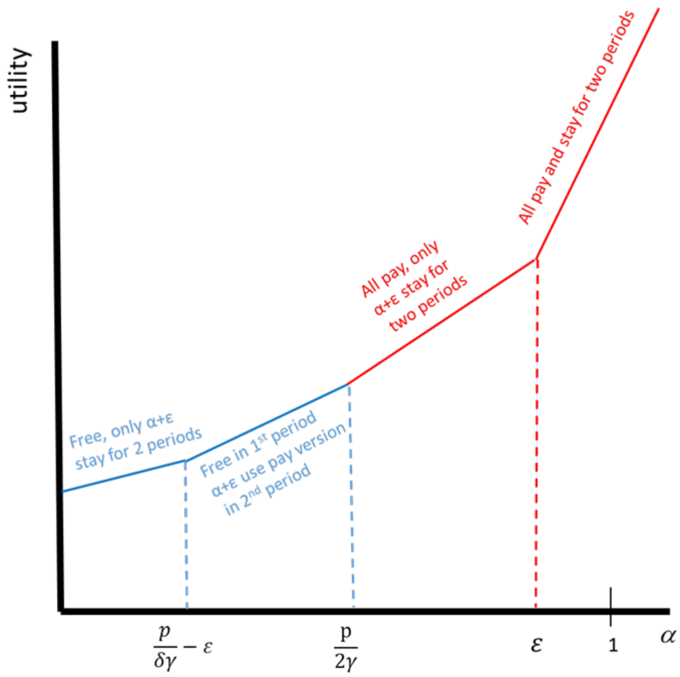
The partial derivatives of the optimal advertising and profit levels are:

$\frac{\partial \gamma}{\partial \varepsilon} = -\frac{k}{4} < 0; \frac{\partial \gamma}{\partial \delta} = 0; \frac{\partial \gamma}{\partial k} = \frac{4-\varepsilon}{4} > 0$

<u>Pay-Based Scenario</u>: Case 2.2 of Appendix A

Observe the following figure (Figure 2 in the text):

Figure B3: Pay-Based Scenario



In this case, when $\alpha < \frac{p-\delta\gamma\varepsilon}{\delta\gamma}$, everyone uses the free version in Period 1, and those with positive utility realization stay for a second period. When $\frac{p-\delta\gamma\varepsilon}{\delta\gamma} < \alpha < \frac{p}{2\gamma}$, everyone uses the free version in Period 1, those with positive realization switch to the paid version in the second period, and those with negative realization churn. When $\frac{p}{2\gamma} < \alpha < \varepsilon$, everyone uses the paid version in Period 1, and those with positive utility realization stay for a second period. Finally, when $> \varepsilon$, everyone uses the paid version for both periods.

Here, $P_1 = 1 - \frac{p}{2\gamma}$; $P_2 = \frac{1}{2}\left(\frac{p}{2\gamma} - \frac{p-\delta\gamma\varepsilon}{\delta\gamma}\right)$; $A_1 = \frac{p}{2\gamma}$; and $A_2 = \frac{1}{2} \cdot \frac{p-\delta\gamma\varepsilon}{\delta\gamma}$

$$P_{12} = 1 - \varepsilon + \frac{1}{2}\left(\varepsilon - \frac{p}{2\gamma}\right) = 1 - \frac{p}{4\gamma} - \frac{\varepsilon}{2}$$

$$\Pi = p\left(1 - \frac{p}{2\gamma}\right) + \frac{p}{2}\left(\frac{p}{2\gamma} - \frac{p - \delta\gamma\varepsilon}{\delta\gamma}\right) + k\gamma\frac{p}{2\gamma} + \frac{k\gamma(p - \delta\gamma\varepsilon)}{2\delta\gamma} - \gamma^2$$

Differentiating with respect to *p*, equating to zero, and collecting terms yield:

$$p_{opt} = \frac{\gamma[k(1+\delta)+\delta(2+\varepsilon)]}{2+\delta};$$

Differentiating with respect to γ, equating to zero, and collecting terms yield:

$$\frac{1}{4}\left[\frac{p^2(2+\delta)}{\gamma^2\delta} - 2k\varepsilon - 8\gamma\right] = 0$$

Substitute the optimal *p* from the above equation, and solve to get the optimal *γ*:

$$\gamma_{opt} = \frac{k^2(1+\delta)^2 + 2k\delta(2+2\delta-\varepsilon) + \delta^2(2+\varepsilon)^2}{8\delta(2+\delta)}$$

Now substitute γ_{opt} into p_{opt} to obtain the solution of optimal price as a function of the parameters.

$$p_{opt} = \frac{[k(1+\delta)+\delta(2+\varepsilon)][k^2(1+\delta)^2 + 2k\delta(2+2\delta-\varepsilon) + \delta^2(2+\varepsilon)^2]}{8\delta(2+\delta)^2}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{([k+(2+k)\delta]^2 - 2(k-2\delta)\delta\varepsilon + \delta^2\varepsilon^2)^2}{64\delta^2(2+\delta)^2}$$

The partial derivatives of the optimal advertising and price levels are:

$$\frac{\partial\gamma}{\partial\varepsilon} = \frac{\delta(2+\varepsilon)-k}{4(2+\delta)} > 0$$

$$\frac{\partial\gamma}{\partial\delta} = \frac{[\delta(2+\varepsilon)-k][k(1+\delta)+\delta(2+\varepsilon)]}{4\delta^2(2+\delta)^2} > 0$$

$$\frac{\partial\gamma}{\partial k} = \frac{(1+\delta)[k+\delta(2+k)] - \delta\varepsilon}{4\delta(2+\delta)} > 0$$

We show that the following three derivatives are positive using numerical simulations, wherein the derivatives are always positive for δ values that match the numerical simulations' solutions, as described in section 4 of the paper.

$$\frac{\partial p}{\partial\varepsilon} = \frac{k^2(\delta^2-1) + 3\delta^2(2+\varepsilon)^2 + 2k\delta[2-\varepsilon+\delta(4+\varepsilon)]}{8(2+\delta)^2} > 0$$

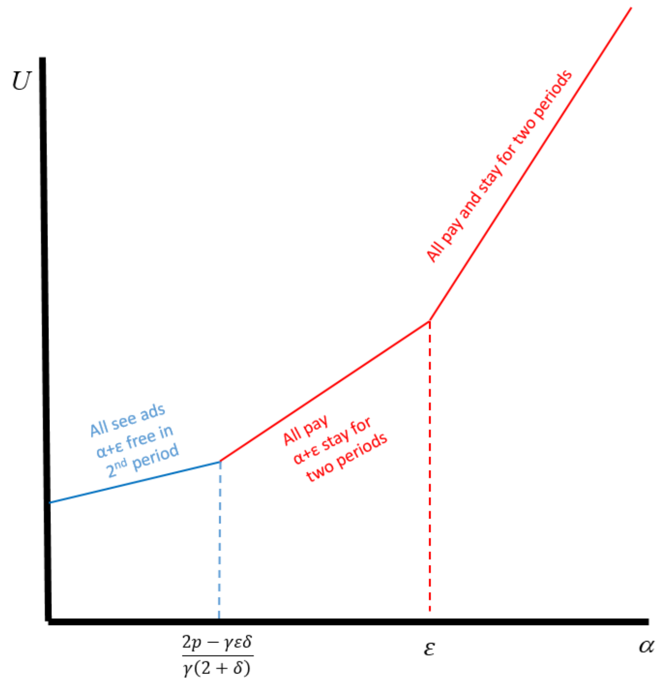
$$\frac{\partial p}{\partial\delta} = \frac{k^3(\delta-2)(1+\delta)^2 + 4\delta^3(2+\varepsilon)^3 + k\delta^2(2+\varepsilon)(12+18\delta-2\varepsilon+5\delta\varepsilon) + 2k^2\delta^2[6+\varepsilon+2\delta(3+\varepsilon)]}{8\delta^2(2+\delta)^3} > 0$$

$$\frac{\partial p}{\partial k} = \frac{3(1+\delta)[k+\delta(2+k)]^2 + 2\delta\varepsilon[2\delta-k+\delta^2(4+k)] + \delta^2\varepsilon^2(\delta-1)}{8\delta(2+\delta)^2} > 0$$

<u>Pay-Based Scenario, special case 1</u>: (Cases 1.1 and 2.1 of Appendix A)

For Figure B3 to hold, we need $\frac{p}{\delta\gamma} - \varepsilon \leq \frac{p}{2\gamma}$ to be true. In this special case, we address the situation in which the reverse holds, as shown in Figure B4.

Figure B4: Pay-Based Scenario, special case 1



In this case, we can combine Cases 1.1 and 2.1 of Appendix A, as they share the same intersection point. In this special case, when $\alpha < \frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)}$, everyone uses the free version in period 1, and those with positive utility realization stay for a second period. When $\frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)} < \alpha < \varepsilon$, everyone uses the paid version in Period 1, and those with positive realization stay for a second period. Finally, when $> \varepsilon$, everyone uses the paid version for both periods.

The profit function components are:

Here, $P_1 = 1 - \frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)}$; $P_2 = 0$; $A_1 = \frac{2p-\gamma\varepsilon\delta}{\gamma(2+\delta)}$; and $A_2 = \frac{2p-\gamma\varepsilon\delta}{2\gamma(2+\delta)}$

$$P_{12} = 1 - \varepsilon + \frac{1}{2}\left(\varepsilon - \frac{2p - \gamma\delta\varepsilon}{\gamma(2+\delta)}\right) = \frac{\gamma(2+\delta-\varepsilon) - p}{\gamma(2+\delta)}$$

$$\Pi = p\left(1 - \frac{2p - \gamma\varepsilon\delta}{\gamma(2+\delta)}\right) + k\gamma\frac{3(2p - \gamma\varepsilon\delta)}{2\gamma(2+\delta)} - \gamma^2$$

Differentiating with respect to *p*, equating to zero, and collecting terms yield:

$p_{opt} = \frac{\gamma}{4}(2 + 3k + \delta + \varepsilon\delta)$;

Differentiating with respect to γ, equating to zero, and collecting terms yield:

$$\frac{2p^2}{\gamma^2(2+\delta)} - \frac{3k\delta\varepsilon}{2(2+\delta)} - 2\gamma = 0$$

Substitute the optimal p from the above equation and solve to get the optimal γ:

$$\gamma_{opt} = \frac{(2+3k+\delta+\delta\varepsilon)^2 - 12k\delta\varepsilon}{16(2+\delta)}$$

Now substitute γ_{opt} into p_{opt} to obtain the solution of optimal price as a function of the parameters.

$$p_{opt} = (2+3k+\delta+\varepsilon\delta)\frac{(2+3k+\delta+\delta\varepsilon)^2 - 12k\delta\varepsilon}{64(2+\delta)}$$

The optimal profits are then:

$$\Pi_{opt} = \frac{(9k^2 + 6k(2+\delta-\delta\varepsilon) + (2+\delta+\delta\varepsilon)^2)^2}{256(2+\delta)^2}$$

The partial derivatives of the optimal advertising, price, and profit levels are:

$$\frac{\partial\gamma}{\partial\varepsilon} = \frac{\delta(2+3k+\delta+\delta\varepsilon) - 6k\delta}{8(2+\delta)} > 0$$

$$\frac{\partial\gamma}{\partial\delta} = \frac{(2-3k+\delta+\delta\varepsilon)(2+3k+\delta+(4+\delta)\varepsilon)}{16(2+\delta)^2} > 0$$

$$\frac{\partial\gamma}{\partial k} = \frac{3(2+3k+\delta-\delta\varepsilon)}{8(2+\delta)} > 0$$

$$\frac{\partial p}{\partial\varepsilon} = \frac{3\delta(-3k^2 + 2k(2+\delta-\delta\varepsilon) + (2+\delta+\delta\varepsilon)^2)}{64(2+\delta)} > 0$$

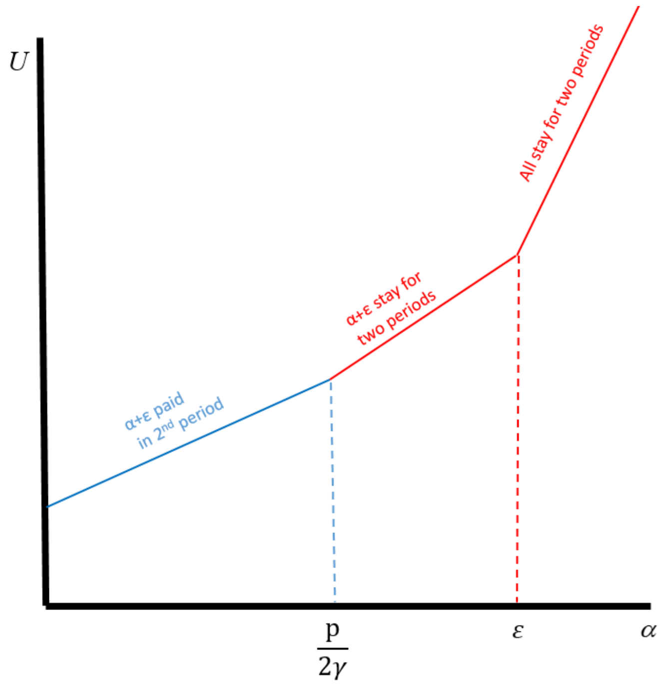
$$\frac{\partial p}{\partial\delta} = \frac{9k(2+\delta)^2 - 18k^2\varepsilon + 6k(2+\delta)^2\varepsilon - 3k\delta(4+\delta)\varepsilon^2 + 2(2+\delta+\delta\varepsilon)^2(2+\delta+(3+\delta)\varepsilon) - 27k^3}{64(2+\delta)^2} > 0$$

$$\frac{\partial p}{\partial k} = \frac{9(2+3k+\delta)^2 + 6\delta(2-3k+\delta)\varepsilon - 3\delta^2\varepsilon^2}{64(2+\delta)} > 0$$

<u>Pay-Based Scenario, special case 2</u>

For Figure B3 to hold, we need $\frac{p}{\delta\gamma} - \varepsilon > 0$ to be true. In this special case, we address the situation in which the reverse holds, as shown in Figure B5. As we show below, this case contradicts the requirement that $k \geq 0$, and therefore does not exist. Assume, *a contrario*, that the intersection does occur at $\alpha = \frac{p}{2\gamma}$ (see Figure B5), and thus when $< \frac{p}{2\gamma}$, everyone uses the free version in Period 1, those with $\alpha + \varepsilon$ switch to the paid version in the second period, and those with negative utility realization churn. When $\frac{p}{2\gamma} < \alpha < \varepsilon$, everyone uses the paid version in Period 1, and those with positive realization stay for a second period. Finally, when $> \varepsilon$, everyone uses the paid version for both periods.

Figure B5: Pay-Based Scenario, special case 2



Here, $P_1 = 1 - \frac{p}{2\gamma}$; $P_2 = \frac{1}{2}\left(\frac{p}{2\gamma}\right)$; $A_1 = \frac{p}{2\gamma}$; and $A_2 = 0$

$$\Pi = p\left(1 - \frac{p}{2\gamma} + \frac{p}{4\gamma}\right) + k\gamma\frac{p}{2\gamma} - \gamma^2$$

Differentiating with respect to p and γ, equating to zero, and collecting terms yield:

$$p_{opt} = (2 + k)\gamma;\ \gamma_{opt} = \sqrt[3]{\frac{p^2}{8}}$$

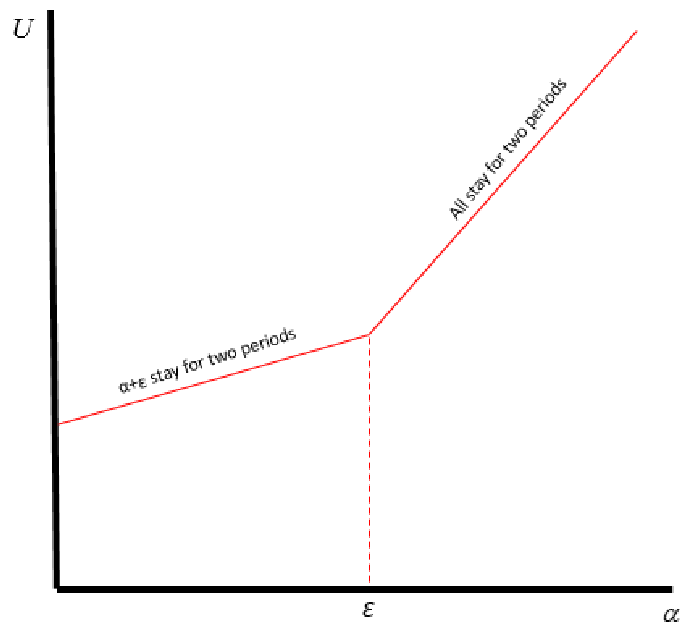
Validating that there are k values that satisfy the constraint on p for this case:

$p_{opt} = (2 + k)\gamma < \gamma\varepsilon\delta$, implying that $k < \varepsilon\delta - 2 < 0$, a contradiction.

<u>Pay-Based Scenario, special case 3</u> In this special case, $p < \varepsilon\gamma\delta/2$ and we address the situation in which $U_p(0) > U_a(0)$. As we show here, this case does not exist, that is, the optimal price does not conform to the requirement of this case.

Assume, *a contrario*, that $p < \varepsilon\gamma\delta/2$. As U_p's slope is always greater than that of U_a, the two lines (paid and free) do not intersect, and everyone pays for the app in Period 1. See Figure B6.

Figure B6: Ad-Based Scenario, special case



In this case, when $\alpha < \varepsilon$, everyone pays in Period 1, and those with positive utility realization stay for a second period. When $\alpha > \varepsilon$, everyone stays for both periods.

Here, $P_1 = 1;\ P_{12} = P_2 = A_1 = A_2 = 0$

$\Pi = p - \gamma^2$

This profit function is linear in price, and so the optimal solution is:

$\gamma_{opt} = 0; p_{opt} = 1$

However, $p < \frac{\varepsilon\gamma\delta}{2} < 1$ contradicts the optimal p.

Web Appendix C: Profits and uncertainty

In this appendix we show the conditions under which profits are increasing under uncertainty (ε).

Pay-Based Scenario (main case)

The optimal profits are given by (see Appendix B):

$$\Pi_{opt} = \frac{([k + (2 + k)\delta]^2 - 2(k - 2\delta)\delta\varepsilon + \delta^2\varepsilon^2)^2}{64\delta^2(2 + \delta)^2}$$

$$\partial\Pi_{opt}/\partial\varepsilon = \frac{2([k + (2 + k)\delta]^2 - 2(k - 2\delta)\delta\varepsilon + \delta^2\varepsilon^2)^1}{64\delta^2(2 + \delta)^2} A$$

Where:

$$A = -2(k - 2\delta)\delta + 2\delta^2\varepsilon = 2\delta(-k + 2\delta + \delta\varepsilon)$$

$\partial\Pi_{opt}/\partial\varepsilon$ is positive if:

$$k < \delta(2 + \varepsilon)$$

Pay-Based Scenario (special case 1)

The optimal profits are given by (see Appendix B):

$$\Pi_{opt} = \frac{(9k^2 + 6k(2 + \delta - \delta\varepsilon) + (2 + \delta + \delta\varepsilon)^2)^2}{256(2 + \delta)^2}$$

$$\partial\Pi_{opt}/\partial\varepsilon = \frac{2(9k^2 + 6k(2 + \delta - \delta\varepsilon) + (2 + \delta + \delta\varepsilon)^2)^1}{256(2 + \delta)^2} B$$

where:

$$B = -6k\delta + 2(2 + \delta + \delta\varepsilon)^1\delta = 2\delta(-3k + 2 + \delta + \varepsilon\delta)$$

$\partial\Pi_{opt}/\partial\varepsilon$ is positive if

$$k < \frac{2 + \delta(1 + \varepsilon)}{3}$$

Numerical simulation of these two cases reveals that in all value parameters of the pay-based scenario (main case), indeed $k < \delta(2 + \varepsilon)$, while in the special case, $k \leq (2 + \delta(1 + \varepsilon))/3$.